Computational Economics (2021) 58:1025–1049
https://doi.org/10.1007/s10614-020-09987-z

Research on the Effects of Institutional Liquidation Strategies on the Market Based on Multi-agent Model

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Accepted: 16 April 2020 / Published online: 2 May 2020

Abstract
Based on the multi-agent model, an artificial stock market with four types of traders is constructed. On this basis, this paper focuses on comparing the effects of liquidation behavior on market liquidity, volatility, price discovery efficiency and long memory of absolute returns when the institutional trader adopts equal-order strategy, Volume Weighted Average Price (VWAP) strategy and Implementation Shortfall (IS) strategy respectively. The results show the following: (1) the artificial stock market based on multi-agent model can reproduce the stylized facts of real stock market well; (2) among these three algorithmic trading strategies, IS strategy causes the longest liquidation time and the lowest liquidation cost; (3) the liquidation behavior of institutional trader will significantly reduce market liquidity, price discovery efficiency and long memory of absolute returns, and increase market volatility; (4) in comparison, IS strategy has the least impact on market liquidity, volatility and price discovery efficiency, while VWAP strategy has the least impact on long memory of absolute returns.

Keywords Multi-agent model · Artificial stock market · Effects of liquidation strategies · Algorithmic trading · Equal-order strategy · VWAP strategy · IS strategy

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1 Introduction

Modern financial markets are highly uncertain. In many cases, some institutional traders need to liquidate their large asset positions within a short period of time. But such quick liquidations usually have notable impacts on asset prices, resulting in high liquidation costs. To effectively reduce transaction costs, traders often liquidate by splitting orders. However, the extension of transaction time will increase time cost and uncertainty risk. Therefore, selecting the effective liquidation strategies to maximize the returns has become an important direction in the current research field of algorithmic trading. At the same time, as increasing numbers of financial institutions adopt algorithmic trading strategies to conduct transactions, the effects of their trading behavior on some aspects of overall market, such as market liquidity, logarithmic returns and volatility, also attract the attention of investors and market regulators. Nevertheless, the performance and the effects on the market of algorithmic trading have not been investigated in depth. In order to test the actual implementation performance of different algorithmic trading strategies and the effects of institutional investors' liquidation behavior on the market when they adopt liquidation strategies, this paper establishes an artificial stock market through the multi-agent model and compares several commonly used algorithmic trading strategies under the same market environment settings. The results may provide important references for the majority of investors and regulators in formulating trading strategies and managing market risks.

Researches on liquidation strategies primarily focus on the impact of liquidation behavior on market price (Bertsimas and Lo 1998; Almgren and Chriss 2001; Almgren 2003; Almgren and Lorenz 2007; Gatheral 2010), the optimal liquidation path based on different risk criteria (Almgren and Chriss 2001; Gokay et al. 2011; Forsyth et al. 2012; Jin 2017), and the application of common algorithmic trading strategies, such as IS strategy (Perold 1988; Hisata and Yamai 2000; Almgren and Chriss 2001; Almgren 2003; Lorenz and Almgren 2011), VWAP strategy (Berkowitz et al. 1988; Konishi 2002; Humphery-Jenner 2011; Frei and Westray 2015), and equal-order strategy or Time Weighted Average Price (TWAP) strategy (Kuno and Ohnishi 2015; Stoikov and Waeber 2016). Since IS strategy, VWAP strategy and equal-order strategy are widely used in academic research and practical application, we will take them as analysis objects in this paper.

At present, the analysis and comparison of the execution performance of algorithmic trading strategies are still insufficient, and only a few literatures provide some ideas and results. Domowitz and Yengerman (2005) found that the performance of algorithmic trading decreased with the increase of order size. Kissell (2007) provided two statistical approaches for the comparison of different algorithmic trading strategies. Besides, in terms of the effects of algorithmic trading on the market, the current works mainly focus on empirical research. Hendershott and Riordan (2009) and Hendershott et al. (2011) studied data from New York Stock Exchange (NYSE) and Deutsche Boerse (DB) respectively and concluded that algorithmic trading was helpful in improving liquidity and price efficiency.

Chaboud et al. (2014) and Viljoen et al. (2014) respectively studied the effects of algorithmic trading on foreign exchange market and futures market. Weller (2017) demonstrated that algorithmic trading might lower price informativeness. Boehmer et al. (2018) analyzed the effects of algorithmic trading on the market in 42 global stock markets from 2001 to 2011, and found that algorithmic trading improved market liquidity and information efficiency and reduced implementation shortfalls, but led to the increase of short-term market volatility.

In the study of optimal liquidation strategy, the unverifiability of its theoretical results is a fatal flaw. Liquidation behavior usually has an impact on the market. But it is difficult to accurately measure the specific impact of different trading strategies on various aspects of the actual market in different market environments through empirical methods. Therefore, it is hardly to verify and compare the actual performance of various liquidation strategies quantitatively. The shortcoming has currently affected traders' investment decisions and made it difficult for market regulators to manage market risks.

The emergence of Agent-Based Computational Finance (ACF) provides a way to solve the above problems. In 1989, the Santa Fe Institute of the United States established the Santa Fe Institute Artificial Stock Market (SFI-ASM), and first used the agent-based model to study the stock market. The establishment of the SFI-ASM model marked the birth of ACF, and the artificial stock market model became an important tool of ACF. ACF models the behavior patterns of the agents from the micro level through the "bottom-up" approach. It can simulate the interaction between the agents and the interaction between the agents and environment more intuitively and explain the emerging macro phenomena from the micro perspective. Representative literatures on this type of model include (Arthur et al. 1997; Brock and Hommes 1998; Bullard and Duffy 1998, 1999; LeBaron et al. 1999; Johnson 2002; Raberto et al. 2003; Noe et al. 2003, 2006; LeBaron 2006; Martinez-Jaramillo 2007; Farmer and Foley 2009; Anufriev et al. 2013; Battiston et al. 2016; Ponta et al. 2018; Dieci and He 2018). Tesfatsion (2003), LeBaron (2006) and Mizuta (2016) then gave some overviews of the multi-agent financial market model.

Compared with the traditional financial theory models, the ACF method virtually liberates the strict assumptions in the traditional analytical framework. Based on the assumption that traders are heterogeneous and bounded rational, the financial market is regarded as a complex system with dynamic evolution, which enables the model to better simulate the real financial market. The simulation results can often reproduce various stylized facts that are hardly explained by traditional financial theories (Lux 1998; LeBaron et al. 1999; Lux and Michele 2000; He and Li 2007; Chiarella et al. 2012). When building an agent-based model, the two most typical types of traders are fundamentalists and chartists (trend followers). He and Li (2007) introduced fundamentalists and chartists to analyze the mechanism generating the power-law distribution fluctuations. Chiarella et al. (2012) conducted a dynamic analysis of a microstructure model based on the continuous double auction mechanism, with heterogeneous agents who can choose to adopt the fundamental strategy or the trend follow strategy according to the strategies' performance. The price sequence formed by the simulation characterizes most of the stylized facts including the volatility clustering, the leptokurtic and fat-tailed distribution and insignificant autocorrelation of returns. In addition, the method based on multi-agent model allows the environment to have feedback on the behavior of the agents. In other words, the agents' trading strategies or transaction behavior will be affected by the environment, and the transaction behavior will affect the environment, in turn. Compared with traditional testing methods, these ACF models with "response" can better help us to study the effects of liquidation strategies on the market.

According to our knowledge, only a few literatures have studied the impacts of algorithmic trading on the market based on the multi-agent simulation market. Gsell (2008) constructed a multi-agent simulation environment based on the weighted behavior model and analyzed two simple algorithmic trading strategies, and the results showed that the execution of large positions implemented by algorithmic trading might make impacts on the market price and volatility. Lee et al. (2011), Brewer et al. (2013) and Brogaard et al. (2017) examined the effects of high-frequency trading (HFT) strategies on financial market by using agent-based model.

In this paper, we conduct an artificial stock market with the continuous double auction mechanism based on the model in Chiarella et al. (2012) to simulate the real stock market. Based on this model, the performance of the algorithmic trading strategies and the effects of liquidation behavior on the market are studied. We find that: (1) the artificial stock market can reproduce stylized facts of real stock market, such as leptokurtosis and fat-tail characteristic of returns and the U-shaped distribution of the intraday trading volume; (2) IS strategy causes the longest liquidation time and lowest liquidation cost; (3) no matter what kind of algorithmic trading strategy is adopted, the liquidation behavior of institutional trader will significantly reduce market liquidity, price discovery efficiency and long memory of absolute returns, and increase market volatility; (4) IS strategy has the least impact on the market liquidity, volatility, and price discovery efficiency; while VWAP strategy has the least impact on the long memory of the absolute returns.

The remainder of the paper is structured as follows. Section 2 establishes an agent-based model framework for an artificial stock market. Section 3 displays basic characteristics of the artificial stock market, and on this basis compares the transaction costs of different liquidation strategies. The effects of institutional liquidations on the market are analyzed in Sect. 4. Finally, we present the study's conclusions in Sect. 5.

2 Model and Methods

Based on the work of Chiarella et al. (2012) and the hypothesis of traders' bounded rationality, we design an artificial stock market (hereinafter referred to as "simulated market") to explore the actual liquidation performance under different algorithmic trading strategies and the influences of liquidation behavior on the overall market. To clearly demonstrate and study these influences separately, we assume that there is only one stock asset being traded in the simulated market and stipulate that traders are not allowed to sell short. Set one minute as a trading time period, denoted by t. The total time of one trading day is 240 min, that is, $t = 1, 2, \dots, 240$. We set the market to be driven by the continuous double auction trading mechanism, which is the main transaction mechanism of stock exchanges in China, including Shanghai Stock Exchange (SSE) and Shenzhen Stock Exchange (SZSE), and also widely adopted in other stock exchanges around world, including NYSE, National Association of Securities Dealers Automated Quotations (NASDAQ), European New Exchange Technology (Euronext) and Australian Securities Exchange (ASX). In such market, the limit orders placed by traders are matched according to the principle of price priority and time priority. The remaining incomplete orders will be stored in the limit order books for later matching. The order books will be cleared at the end of each trading day. The simulated market that we set primarily includes the fundamentalists who adopt the fundamental strategy and the chartists who adopt the trend follow strategy. In addition, to guarantee the liquidity of the market, we also introduce random traders in the simulated market.

Different from the artificial stock market model designed by Chiarella et al. (2012), on the one hand, we add an institutional trader into the basic model composed of three types of heterogeneous traders, i.e. the fundamentalists, the chartists and the random traders. The institutional trader holds a large amount of positions and can adopt different algorithmic trading strategies to liquidate them. On the other hand, we design the distribution of trading volume based on the stylized facts in the real stock market. At the beginning of each trading day, as the traders usually have relatively adequate information, they are more willing to enter the market for trading. With the consumption of information, the closer to the middle of 1 day's trading period, the less information traders have, and the less willing to trade they are. As the end of the trading period approaches, the information acquired by traders accumulates again, which leads to a renewed increase in their willingness to trade. Therefore, the U-shaped distribution of intraday trading volume is formed. To make the simulated market as close as possible to the distribution of trading volume in the real stock market, we describe the activity of traders using a quadratic convex function. In other words, the closer to the beginning and end of the trading time of days, the more active traders will be, and the more people will enter the market for trading; however, the closer to noon, the less active traders will be, and the fewer people will enter the market for trading. At the same time, we set the traders' entry order for each time period to be random.

2.1 Heterogeneous Traders and Their Trading Strategies

2.1.1 Fundamentalists

Fundamentalists focus on the fundamental value of the stock and make trading decisions based on the difference between the market price of the stock and its fundamental value. The fundamentalists will choose to buy the stock when they believe it is undervalued compared to its fundamental value and otherwise they will choose to sell. Suppose the evolution formula of the fundamental value of the stock is as follows:

$$p_{t+1}^* = p_t^* \cdot \exp\left(\sigma^f \cdot \epsilon_t^f\right) \tag{2.1}$$

where $t(t = 1, 2, \dots)$ is the trading time period, $\sigma^f > 0$ is the volatility of the fundamental value, and ϵ_t^f is a random variable obeying the standard normal distribution. Assume that the time when the fundamentalist enters the market is $\tau \in [t, t + 1)$, and the latest transaction price of the stock $p_{t,\tau}$ is the current market price of the stock. The probability that the fundamentalist submits the order at this time $s_{t,\tau}^f$ is proportional to the difference between the fundamental value p_t^* and the latest transaction price of the stock $p_{t,\tau}$:

$$s_{t,\tau}^f = \min(1, a^f \cdot |p_t^* - p_{t,\tau}|) \tag{2.2}$$

where $a^f > 0$ indicates the sensitivity of the fundamentalist's trading willingness to the difference between the fundamental value and the market price. The larger the a^f is, the more sensitive to the price difference between the two, and the more likely it is to submit an order in the presence of the spread. We set the direction of submitting order $d_{t,\tau}^f$ as

$$d_{t,\tau}^f = \operatorname{sgn}\left(p_t^* - p_{t,\tau}\right) \tag{2.3}$$

when $d_{t,t}^f = 1$ and the trader's cash can afford at least 1-unit stock position, he/she will submit the buy order. When $d_{t,\tau}^f = -1$ and the position held by the trader is not less than 1-unit, he/she will submit the sell order. Consistent with the assumption of Chiarella et al. (2012), we assume that the price of the order submitted by the trader $\tilde{p}_{t,\tau}^{\mathrm{f}}$ is subject to a uniform distribution between the current fundamental value p_t^* and the market price $p_{t,\tau}$.

$$\tilde{p}_{t,\tau}^{\mathrm{f}} = \begin{cases} U\left(p_{t,\tau}, p_t^*\right) & p_{t,\tau} < p_t^* \\ U\left(p_t^*, p_{t,\tau}\right) & p_{t,\tau} > p_t^* \end{cases} \tag{2.4}$$

Let the order volume submitted by the fundamentalist be

$$\tilde{q}_{t,\tau}^f = \begin{cases} \min\left(\frac{cash_{t,\tau}^i}{\tilde{p}_{t,\tau}^f}, U\left(1, \theta \cdot \frac{cash_{t,\tau}^i}{\tilde{p}_{t,\tau}^f}\right) \cdot \left[1 + \frac{p_t^* - p_{t,\tau}}{p_t^*}\right]\right) & d_{t,\tau}^f = 1 \\ \min\left(h_{t,\tau}^i, U\left(1, \theta \cdot h_{t,\tau}^i\right) \cdot \left[1 + \frac{p_{t,\tau} - p_t^*}{p_{t,\tau}}\right]\right) & d_{t,\tau}^f = -1 \end{cases} \tag{2.5}$$

where $cash_{t,\tau}^i$ and $h_{t,\tau}^i$ are the amount of cash and the number of positions held by the i-th trader, respectively, and $\theta > 0$ is the intensity of submitting orders. It can be observed from the formula that the composition of the fundamentalist's order volume is mainly the product of two parts. The first part is subject to a uniform distribution between 1 and the maximum volume of orders can be submitted ($cash_{t,\tau}^i / \tilde{p}_{t,\tau}^{\mathrm{f}}$ or $h_{t,\tau}^i$), multiplied by the intensity of submitting orders. The maximum volume of orders that can be submitted determines that the trader's order volume is limited by his/her own wealth. In the second part, we consider the difference between the fundamental value and the market price and use it as an amplifier. In other words, the order volume is proportional to the difference between the fundamental value and the market price. At the same time, we also place a limit such that the order volume will not exceed its maximum volume of orders that can be submitted ($cash^i_{t,\tau}/\tilde{p}^{\mathrm{f}}_{t,\tau}$ or $h^i_{t,\tau}$).

2.1.2 Chartists

Chartists use the stock's moving average prices to predict future stock movements and make trading decisions. When the current market price of the stock is higher than the moving average price, the chartists believe that the stock will continue to rise and they choose to buy and otherwise they will choose to sell. The moving average price of the stock price determined by the i-th chartist $mp^{L^i}_t$ is set as

$$mp^{L^i}_t = \frac{1}{L^i} \cdot \sum_{j=1}^{L^i} p^{close}_{t-j} \tag{2.6}$$

where p^{close}_{t-j} is the closing price at time $t-j$, and L^i is the length of the time window observed by the i-th trader. The probability of the chartist's order submission is

$$s^c_{t,\tau} = \left|\tanh\left(a^c \cdot \psi^{L^i}_{t,\tau}\right)\right| \tag{2.7}$$

where $a^c > 0$ is a constant indicating the aggressiveness of the trader's order submission, and $\psi^{L^i}_{t,\tau} = p_{t,\tau} - mp^{L^i}_t$ is the difference between the market price and the moving average price. We consider the direction of submitting order $d^c_{t,\tau}$ as

$$d^c_{t,\tau} = sgn\left(\psi^{L^i}_{t,\tau}\right) \tag{2.8}$$

when $d^c_{t,\tau} = 1$ and the trader's cash can afford at least 1-unit stock position, submit the buy order. When $d^c_{t,\tau} = -1$ and the position held by the trader is not less than 1-unit, he/she will submit the sell order.

Suppose that the price of submitting order $\tilde{p}^{\mathrm{c}}_{t,\tau}$ is based on the current market price $p_{t,\tau}$:

$$\tilde{p}^{\mathrm{c}}_{t,\tau} = p_{t,\tau} \cdot \left(1 + \sigma^c \cdot \varepsilon^c_{t,\tau}\right) \tag{2.9}$$

where $\sigma^c \geq 0$ denotes the deviation range between the price of submitting order and market price, and $\varepsilon^c_{t,\tau} \sim N(0,1)$ obeys the standard normal distribution.

Let the order volume submitted by the chartist be

$$\tilde{q}^c_{t,\tau} = \begin{cases} U\left(1, \theta \cdot \frac{cash^i_{t,\tau}}{\tilde{p}^c_{t,\tau}}\right) & d^c_{t,\tau} = 1 \\ U\left(1, \theta \cdot h^i_{t,\tau}\right) & d^c_{t,\tau} = -1 \end{cases}. \tag{2.10}$$

In other words, the volume obeys the uniform distribution between 1 and its maximum volume of orders can be submitted multiplied by the intensity of submitting orders.

2.1.3 Random traders

To ensure market liquidity, we also introduce random traders in the model. The direction of submitting order of the random trader is randomly set to bid and ask. The price of submitting order is $\tilde{p}^{\mathrm{r}}_{t,\tau} = p_{t,\tau} + \sigma^r \varepsilon^r_{t,\tau}$, where $\sigma^r \geq 0$ denotes the deviation range between the price of submitting order by the random trader and market price, and $\varepsilon^r_{t,\tau} \sim N(0, 1)$ obeys the standard normal distribution. Similarly, the order volume submitted by the random trader $\tilde{q}^{\mathrm{r}}_{t,\tau}$ is limited by the maximum volume of orders that can be submitted, thus

$$\tilde{q}^{\mathrm{r}}_{t,\tau} = \begin{cases} U\left(1, \theta \cdot \frac{cash^i_{t,\tau}}{\tilde{p}^r_{t,\tau}}\right) & d^r_{t,\tau} = 1 \\ U\left(1, \theta \cdot h^i_{t,\tau}\right) & d^r_{t,\tau} = -1 \end{cases}. \tag{2.11}$$

2.2 Institutional Trader and Liquidation Strategies

As we all know, institutional traders usually have large positions and needs to clean them through liquidation strategies in limited time periods. In this paper, an institutional trader is introduced into the simulated market, and he/she can adopt three kinds of commonly used algorithmic trading strategies, namely, equal-order strategy, VWAP strategy, and the IS strategy to liquidate the position.

2.2.1 Equal-Order Strategy

Equal-order strategy is the simplest traditional algorithmic trading strategy, which is actually known as TWAP strategy. This strategy divides the trading time evenly, and divides the total position to be liquidated equally for submission according to the number of division nodes, that is, the quantity of orders submitted in each time period is

$$n = \frac{X}{N} \tag{2.12}$$

where X is the total position and N is the number of subintervals of the trading period.

In this paper, the one-day trading period is divided into 240 equal parts in minutes, i.e. $N = 240$.

2.2.2 VWAP Strategy

The VWAP strategy is a trading strategy that breaks up large positions and executes them in batches within an agreed period of time, targeting making the actual weighted average transaction price close to the weighted average price of the whole market volume in this period of time. The VWAP formula is as follows:

$$\mathrm{VWAP} = \frac{\sum_{k=1}^{N} x_k \cdot p_k}{\sum_{k=1}^{N} x_k} = \sum_{k=1}^{N} w_k \cdot p_k \tag{2.13}$$

where $w_i = x_i \Big/ \sum_{k=1}^{N} x_k$ is the weight of the transaction volume x_i traded at price p_i for the total transaction volume $\sum_{k=1}^{N} x_k$.

This paper estimates the distribution of the volume per minute within the trading day to be liquidated by counting the average of the percentage of the volume per minute in the total trading volume on its day in historical data $\left\{\bar{w}_i, i = 1, 2 \ldots 240\right\}$. The total amount of stock position X that needs to be liquidated on that day is split according to this distribution to obtain suborders submitted every minute $\left\{X * \bar{w}_i\right\}$.

2.2.3 IS Strategy

According to Perold (1988)'s research results, the implementation shortfall is the difference between the realized value and the target value determined before the transaction, and the IS strategy makes trading decisions with the goal of minimizing the implementation shortfall. By referring to the method of Hisata and Yamai (2000), we assume that the stock price obeys the arithmetic random walk process, consider the permanent and temporary impacts generated by the trading behaviors in the market and obtain the liquidation strategy under the VaR framework.

Specifically, suppose a stock asset with a total position of X needs to be liquidated within time T and divide T into N subintervals with a time interval of $\Delta t = t_k - t_{k-1} (k = 0, 1, \ldots, N)$. The total time required for liquidation is satisfied: $T = N \cdot \Delta t$. Let the number of shares sold between t_{k-1} and t_{k} be $n_{k-1,k} = x_{k-1} - x_k$, where x_k is the remaining position held at time t_k.

According to the general research methods, this paper divides market impact into permanent impact and temporary impact. The permanent impact will have a lasting impact on the stock price, while the temporary impact will only affect the current price. Assume that there is a linear correlation between the permanent impact and the number of shares sold $n_{k-1,k}$, and γ is the permanent impact coefficient. Thus, the market price containing the permanent impact is

$$P_k = P_{k-1} + \sigma \cdot \Delta t^{\frac{1}{2}} \cdot \varepsilon_k + \mu \cdot \Delta t - \gamma \cdot n_{k-1,k} \tag{2.14}$$

where σ is the volatility of the stock price, μ is the drift rate of the price, and $\varepsilon_k \sim N(0, 1)$.

Suppose the temporary impact and the selling speed of the stock $\frac{n_{k-1,k}}{\Delta t}$ show a linear correlation. The stock price containing temporary impact can be written

$$\tilde{P}_k = P_k - \eta \cdot \frac{n_{k-1,k}}{\Delta t} \tag{2.15}$$

where η is the temporary impact coefficient.

Then, we can obtain the execution cost as

$$C = X \cdot P_0 - \sum_{k=1}^{N} n_{k-1,k} \cdot \tilde{P}_k. \tag{2.16}$$

According to the research results of Bertsimas and Lo (1998), we consider the way of even liquidation, that is $n_{k-1,k} = X/N$, and substitute (2.15) into formula (2.16). Finally, we can obtain

$$E[C] = -\frac{1}{2}\mu \cdot \Delta t \cdot X \cdot (N-1) + \frac{1}{2} \cdot \gamma \cdot X^2 + \frac{\eta \cdot X^2}{\Delta t \cdot N} + \frac{\gamma \cdot X^2}{2 \cdot N} \tag{2.17}$$

$$V[C] = \frac{1}{3}\sigma^2 \cdot \Delta t \cdot X^2 \cdot N \cdot \left(1 - \frac{1}{N}\right) \cdot \left(1 - \frac{1}{2N}\right). \tag{2.18}$$

Based on the research results of Hisata and Yamai (2000), we determine the value of N by minimizing the maximum expected value of the execution cost C under a certain confidence level, namely, the value at risk (VaR) of the execution cost:

$$\min_N VaR_\alpha(N) = \min_N \left\{ E[C] + Z_\alpha \cdot \sqrt{V[C]} \right\} \tag{2.19}$$

where Z_α is the α-quantile of the normal distribution. In the simulation, we set $\alpha = 0.01$. Hisata and Yamai (2000) have shown that, when Δt is small, we can use the explicit solution under continuous time to approximate the optimal solution under discrete time, and the solution is

$$T^* = \left[\frac{2\sqrt{3} \cdot \eta \cdot X}{Z_\alpha \cdot \sigma} \right]^{\frac{2}{3}}. \tag{2.20}$$

The number of optimal suborders finally obtained is

$$N^* = \frac{T^*}{\Delta t} = \left[\frac{2\sqrt{3} \cdot \eta \cdot X}{Z_\alpha \cdot \sigma} \right]^{\frac{2}{3}} \Big/ \Delta t. \tag{2.21}$$

Table 1 Parameter settings in simulation

Parameter	Value	Instruction
P_0	500	Initial stock price
h_0	10,000	Trader's initial position
$cash_0$	5000,000	Trader's initial cash
σ^f	0.0015	Volatility of fundamental value
a^f	0.05	Probability coefficient of fundamentalists to submit orders
a^c	0.05	Probability coefficient of chartists to submit orders
L_i	{20, 21…100}	Time window length for chartists to observe historical data
σ^c	0.01	Range of which the prices of orders submitted by chartists deviate from the market price
σ^r	$\sigma^f \cdot P_0$	Range of which the prices of orders submitted by random traders deviate from the market price
θ	10%	Intensity of submitting orders

3 Basic Characteristics of the Simulated Market and Transaction Costs of Institutional Liquidation

3.1 The Simulated Market without Institutional Trader

To verify the effectiveness of the model, we first simulate the basic model and verify whether the simulation results conform to stylized facts including the leptokurtic and fat-tailed distribution of logarithmic returns and the U-shaped distribution of intraday volume. In the base model, we add 500 fundamentalists, 300 chartists and 200 random traders. The remaining parameter settings in the model are shown in Table 1.

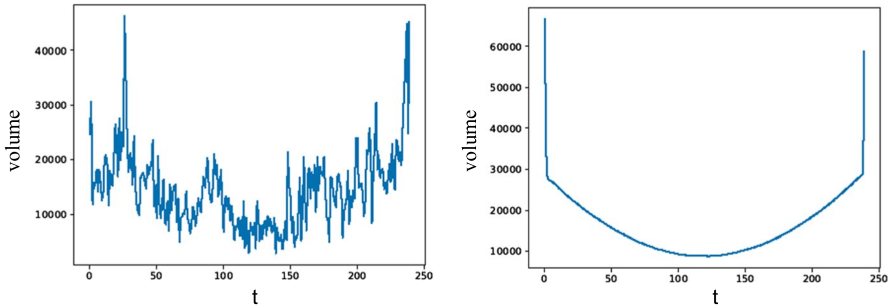
Table 2 Statistics of the returns of simulated market and 29 stocks in SSE 50

	Mean	Std	Kurtosis	Skewness	Fat-tail index
Simulation results					
Mean	4.48E−07	3.54E−03	24.1041	0.0078	0.7675
Std	2.19E−05	2.23E−04	11.2653	0.2737	0.0323
Max	7.13E−05	4.35E−03	147.8428	1.0262	0.8689
Min	− 6.30E−05	2.90E−03	9.5306	− 1.9866	0.6685
Median	4.48E−07	3.54E−03	24.1041	0.0078	0.7675
Empirical results					
Mean	7.83E−06	2.34E−03	27.3490	0.3012	0.3001
Std	5.38E−06	4.67E−04	11.2764	0.2955	0.2011
Max	2.17E−05	3.27E−03	60.9618	0.8228	0.8288
Min	− 1.19E−06	1.00E−03	14.2979	− 0.4699	0.0019
Median	7.83E−06	2.34E−03	27.3490	0.3012	0.3001

We generate stock data through the simulated market with the window of 20 trading days, repeat it 1000 times, and calculate the logarithmic returns' mean, standard deviation, kurtosis, skewness and fat-tail index (see "Appendix 1"). As a comparison, we select the minute data of 29 constituent stocks in the SSE 50 index in 2015 (see Table 7 in "Appendix 2") and calculate the above indicators. The SSE 50 index is a sample stock composed of 50 most representative stocks with large scale and good liquidity in SSE, which can comprehensively reflect the overall situation of the most influential group of leading enterprises in Shanghai stock market. Considering the integrity and availability of data, we select 29 constituent stocks in SSE 50 index, each of which contains more than 55,000 pieces of minute data.

The simulation and empirical results are shown in Table 2. It can be observed that the mean value, standard deviation, kurtosis and skewness of returns in both the simulation results and the empirical results are very close. Furthermore, the mean value of returns in both sets of results is close to 0; the kurtosis is significantly greater than 3; and the fat-tail index is significantly less than 2, indicating that both the simulation results and the empirical results show obvious leptokurtic and fat-tailed characteristics. It should be noted that to verify the robustness of the simulated market, we also show the results of the simulation window of 10 days, 30 days, 40 days and 50 days in Table 8 in "Appendix 2". The results show that the above indicators do not change significantly with the change of simulation days, indicating that the simulation results are stable and our model is reasonably designed, which can well simulate the real stock market. Therefore, the model can be further used as a basic model to study the performance of different liquidation strategies and the effects of liquidation behavior on the market.

In addition, we analyze the simulation data of intraday trading volumes and show the results in Fig. 1. Figure 1a shows the distribution of the trading volume per minute of the stock in a certain trading day. The daily average trading volume per minute distribution for 1000 simulation results in 20 trading days is shown



(a) Schematic diagram of the time distribution of trading volume (1 trading day)

(b) Schematic diagram of the time distribution of the trading volume (average of 1000 simulation results in the window of 20 trading days

Fig. 1 Schematic diagram of volume–time distribution

Table 3 Cost ratios under the three liquidation strategies

Cost rate	Equal-order (%)	VWAP (%)	IS (%)
Mean	0.35	0.30	0.13
Std	1.45	1.40	2.14
Max	5.37	5.07	7.91
Min	− 5.82	− 5.16	− 8.17
Median	0.38	0.31	0.18

Table 4 Significance test of cost ratios under the three liquidation strategies (*p*-value)

	Equal-order	VWAP	IS
Equal-order	–	0.4037	0.0064
VWAP	0.4037	–	0.0354
IS	0.0064	0.0354	–

in Fig. 1b. As can be observed in Fig. 1b, the simulated intraday trading volume presents a typical U-shaped distribution.

3.2 Transaction Costs of Institutional Liquidation

In this section, we introduce an institutional trader based on the basic model of the simulated market above and conduct the simulation experiment with the window of 40 trading days. Assume that the institutional trader needs to sell all of the 2 million positions in a certain period of time, with the total number of position being recorded as H_A. Before liquidation, the institutional trader first observes the stock market operation in the first 20 days and begins liquidating on the 21st trading day. According to the analysis of historical data, the institutional trader uses the equal-order strategy, VWAP strategy and IS strategy to liquidate respectively. During the liquidation process, the institutional trader submits suborders in the form of market orders.

Under the equal-order strategy, the institutional trader splits the large order into 240 equally sized small orders corresponding to 240 min in one trading day, and then submits these suborders for liquidation by minute on the 21st trading day. Under the VWAP strategy, the institutional trader splits the total position into 240 suborders according to the distribution of the minute trading volume on the 21st day, which is estimated by the volume distributions in the first 20 days, and then submits the suborders for liquidation in every minute on that day. Under the IS strategy, we consider Δt equals 1, which means that the institutional trader submits suborders each minute, and the number of optimal suborders N^* can be determined as the description of the IS strategy component in Sect. 2.2. Then the volume of suborders under the optimal liquidation strategy is H_A/N^*. The temporary impact coefficient η and the volatility σ are obtained by the method proposed by Glosten and Lawrence (1988) and the calculation method of realized volatility, respectively, based on the stock data of 20 days before liquidation.

To compare the liquidation costs under different algorithmic trading strategies, we define the liquidation cost ratio as $C_L = \frac{(C_{EL} - C_{RL})}{C_{EL}} \times 100\%$, where C_{EL} is the expected liquidation return (the product of the market price and the position to be liquidated at the beginning of liquidation), and C_{RL} is the actual liquidation return (the actual cash received after liquidation).

The cost rates of the three liquidation strategies adopted by institutional trader and the independent sample t test among them are shown in Tables 3 and 4. The results show that IS strategy has the most remarkable effect in reducing cost, and its average cost rate is 0.13%. And since the p-values of the independent sample t-test with equal-order strategy and VWAP strategy are 0.0064 and 0.0354 respectively, IS strategy is statistically significantly better than them in terms of liquidation cost. Although the average cost rate of the VWAP strategy (0.30%) is better than that of the equal-order strategy (0.35%), there is no statistically significant difference between the two strategies (the p-value is 0.4037). On the other hand, the standard deviation of the cost rate is the highest under the IS liquidation strategy, which is 2.14%. Specifically, the mean of the optimal liquidation days of the IS liquidation strategy in 1000 simulations is 2.32 days (557 min), and the median is 2.39 days (574 min), which is generally longer than the one day liquidation time under the equal-order strategy and the VWAP strategy. This finding suggests that the longer the liquidation takes, the greater the market uncertainty that traders encounter, and the volatility of the liquidation costs will also expand.

4 Effects of Institutional Trader's Liquidation on the Market

In order to analyze the effects of institutional trader's liquidation behavior on the entire market, we introduce several market indicators in aspects of market liquidity, volatility, price discovery efficiency and long memory of absolute returns. We then compare the market qualities when there is no institutional trader's liquidation and when the institution liquidates under three different strategies.

4.1 Market Indicators

4.1.1 Liquidity

Liquidity is the ability of an asset to be quickly liquidated at a reasonable price. We introduce three indicators to measure market liquidity: order transaction ratio PV_t, daily turnover rate TR_d and relative bid-ask spread PR_t. The calculation formulas are as follows:

$$PV_t = V_t / Q_t \tag{4.1}$$

$$TR_d = V_d / CE \times 100\% \tag{4.2}$$

$$PR_t = 2 \times \left(p_{at} - p_{bt}\right) / \left(p_{at} + p_{bt}\right) \tag{4.3}$$

where V_t is the trading volume per minute, Q_t is the volume of orders submitted per minute, V_d is the daily trading volume, CE is the circulating stock capital, p_{at} is the best ask price on order book at the end of each minute, and p_{bt} is the best bid price. Specifically, the higher the order transaction ratio and turnover rate are (or the smaller the relative bid-ask spread is), the better market liquidity is.

4.1.2 Volatility

Volatility is an important indicator to measure the level of market risk. This paper uses historical volatility and relative volatility to measure the volatility of the market. The historical volatility is

$$ER^2 = \frac{1}{RT - 1} \sum_{t=1}^{RT} \left(r_t - \bar{r}\right)^2 \tag{4.4}$$

where RT represents the selected time window when calculating volatility, r_t represents the logarithmic return at each minute, and $\bar{r}$ represents the average value of $\{r_t\}$. Additionally, the relative volatility is

$$RR = 2 \times (HP - LP)/(HP + LP) \tag{4.5}$$

where HP and LP are the highest and lowest transaction prices, respectively, within a day.

4.1.3 Price Discovery Efficiency

The price discovery efficiency measures whether the market price sufficiently reflects the information about fundamental value. Generally, the more liquid and transparent the market is, the more efficient the price discovery of the market will be. To measure the efficiency level of price discovery in the market, we construct the price deviation indicator by dividing the absolute value of the difference between the actual market price and the fundamental value by the fundamental value.

$$PL = \frac{\left|p_t - p_t^*\right|}{p_t^*} \tag{4.6}$$

Hence, the smaller the PL is, the higher the price discovery efficiency of the market is.

4.1.4 Long Memory of Absolute Returns

We use the Hurst exponent to measure whether the time series of absolute returns has characteristics of long memory and use the R/S analysis method first proposed by Mandelbrot (1963) to calculate it. The specific steps of the R/S analysis method are as follows.

For a time series $\{y_t\}$ of length T, the sequence is divided into M equal length subsequences whose length is n, that is, $T = M \times n$. Let $y_{m,i}$ be the i-th element in the subsequence $m(m = 1, 2, \dots, T)$, with the mean of $\bar{Y}_m = \frac{1}{n}\sum_{i=1}^{n} y_{m,i}$ and the standard deviation of $S_m = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_{m,i} - \bar{Y}_m\right)^2}$. The cumulative dispersion of the i-th element in the subsequence m is $Y_{m,i} = \sum_{j=1}^{i}\left(y_{m,j} - \bar{Y}_m\right)$, and the range of the subsequence m can be obtained as $R_m = max\left(Y_{m,i}\right) - min\left(Y_{m,i}\right)$. Then, R_m/S_m is the remark range of subsequence m, and the remark range of the sequence with length n is

$$(R/S)_n = \frac{1}{M}\sum_{m=1}^{M}\left(R_m/S_m\right). \tag{4.7}$$

The following empirical formula should be satisfied for the remark range and the sequence length.

$$(R/S)_n = Cn^H \tag{4.8}$$

Take the logarithm of both sides of the above formula.

$$ln\left((R/S)_n\right) = ln(C) + H \cdot ln(n) \tag{4.9}$$

When the original sequence is segmented, the corresponding $(R/S)_n$ is obtained according to the different values of n; then the OLS of n is conducted to obtain the value H of the Hurst exponent.

4.2 Comparison of Institutional Liquidation Results

4.2.1 Effects of Liquidations on The Market

We first compare the order transaction ratio, turnover rate, relative bid-ask spread, historical volatility, relative volatility, *PL* and Hurst exponent of the market with and without an institutional trader's liquidation. The statistical results of indicators are shown in Table 5.

As can be observed in Table 5, compared to the situation without an institution's liquidation, during institutional liquidation, (1) in terms of the liquidity indicator, the market transaction ratio and the turnover rate are reduced and the market bid-ask spread is expanded, indicating that the market liquidity worsens and the institutional liquidation behavior has consumed market liquidity to a certain extent; (2) in terms of the volatility indicator, both the historical volatility and the relative volatility of the market are significantly increased, indicating that institutional liquidation behavior significantly increases the market volatility; (3) in terms of price discovery efficiency, the *PL* value has been expanded from 0.028 in the market without institutional liquidation to 0.064, 0.0058 and 0.0043 respectively, indicating that the price discovery efficiency has been significantly reduced in the market with institutional

Table 5 Comparison of the indicators with or without institutional liquidation

		Ord. trans. rat. (%)	Turn. rate (%)	Bid. spr.	Hist. vol.	Rel. vol.	PL	Hur. exp.
No Liqu.	Mean	24.72	38.45	0.0016	1.25E−05	0.0560	0.0028	0.6234
	Std	0.12	0.80	0.0001	1.59E−06	0.0035	0.0001	0.0506
	Max	25.25	41.20	0.0019	1.89E−05	0.0680	0.0031	0.7937
	Min	24.37	35.74	0.0014	8.41E−06	0.0459	0.0025	0.4878
	Median	24.70	38.43	0.0016	1.24E−05	0.0558	0.0028	0.6197
Equal-order	Mean	21.70	33.21	0.0050	4.46E−05	0.0717	0.0064	0.4589
	Std	0.40	3.20	0.0005	2.95E−05	0.0234	0.0019	0.0697
	Max	23.71	55.00	0.0072	4.73E−04	0.4253	0.0520	0.6638
	Min	20.49	23.43	0.0035	6.24E−06	0.0321	0.0036	0.2400
	Median	21.70	33.16	0.0050	3.94E−05	0.0683	0.0063	0.4584
VWAP	Mean	22.18	33.06	0.0045	4.35E−05	0.0708	0.0058	0.4749
	Std	0.37	2.97	0.0004	3.25E−05	0.0215	0.0013	0.0703
	Max	23.77	44.41	0.0059	6.65E−04	0.3427	0.0201	0.7100
	Min	20.92	25.41	0.0030	5.39E−06	0.0263	0.0029	0.2717
	Median	22.18	32.95	0.0044	3.75E−05	0.0677	0.0057	0.4761
IS	Mean	23.71	33.15	0.0030	3.66E−05	0.0708	0.0043	0.4563
	Std	0.84	2.53	0.0007	3.30E−05	0.0180	0.0017	0.0538
	Max	25.74	46.15	0.0070	6.57E−04	0.2656	0.0324	0.6454
	Min	17.95	24.59	0.0018	5.26E−06	0.0318	0.0023	0.3054
	Median	23.87	33.16	0.0028	3.08E−05	0.0683	0.0040	0.4543

liquidation; (4) in terms of the long memory of absolute returns, the Hurst exponent decreases obviously, indicating that the liquidation behavior of the institution significantly weakens the long memory of absolute returns.

Finally, we conduct an independent sample t-test on the above statistical results to verify whether there are significant differences in market indicators between institutional liquidation and no-liquidation. The *p*-values of independent sample t-test of pairwise comparison of different situations in each market indicator are shown in Table 6.

As can be observed in Table 6, the *p*-values of the test on all market indicators with and without institutional liquidation are 0.000 (see the first to the third row in Table 6). This finding shows that no matter which kind of liquidation strategy is adopted, each indicator of the market changes significantly after the institution takes liquidation behavior. Specifically, institutional liquidation significantly reduces market liquidity, price discovery efficiency and long memory of absolute returns, and increases market volatility.

4.2.2 Comparison of Effects Under Different Liquidation Strategies on the Market

Finally, on the basis of Tables 5 and 6, we can further compare the differences in the effects of liquidation behaviors under different liquidation strategies on the market.

Table 6 Significance test of the indicators under different market conditions (p-value)

	Ord. trans. ratio	Turnover rate	Bid-ask spread	Historical volatility	Relative volatility	PL	Hurst exponent
No-liqu. and Equ.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
No-liqu. and VWAP	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
No-liqu. and IS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Equ. and VWAP	0.0000	0.3031	0.0000	0.4163	0.3384	0.0000	0.0000
Equ. and IS	0.0000	0.6616	0.0000	0.0000	0.3509	0.0000	0.3629
VWAP and IS	0.0000	0.4876	0.0000	0.0000	0.9173	0.0000	0.0000

In terms of market liquidity, regarding the two indicators of order transaction ratio and bid-ask spread, when the institutional investor uses IS strategy for liquidation, the average value of order transaction ratio is the highest (23.71%) and the average value of bid-ask spread is the lowest (0.0030). Therefore, the IS strategy performs best, the VWAP strategy follows (22.18% and 0.0045, respectively), and the equal-order strategy performs the worst (21.70% and 0.0050, respectively). According to the results in Table 6, the three strategies show statistically significant differences in order transaction ratio and bid-ask spread, indicating that IS can reduce the consumption of market liquidity caused by liquidation behavior to the greatest extent among them. In terms of turnover rate, although there are differences in the average turnover rate of the three strategies, the differences are not statistically significant among the three. Hence, there is no significant difference in the effect of the three liquidation strategies on the market turnover rate indicator. In general, IS strategy performs best in reducing the effect on market liquidity.

In terms of market volatility, the average historical volatility under IS strategy is the lowest (3.66E−05), followed by VWAP (4.35E−05), and equal-order strategy is the highest (4.46E−05), showing statistically significant differences from Table 6. On the relative volatility indicator, although the average relative volatilities (both 0.0708) under VWAP strategy and IS strategy are better than that under equal-order strategy (0.0717), there is no statistically significant difference between the three strategies. In general, compared with the other two strategies, IS strategy can minimize the impact of liquidation behavior on market volatility and ensure the stability of the market to the greatest extent.

In terms of the price discovery efficiency, the average value of *PL* under IS strategy is the lowest (0.0043), followed by VWAP (0.0058), and equal-order strategy is the highest (0.0064), and they show significant differences in statistics from Table 6. This illustrates that IS strategy can minimize the impact of liquidation behavior on market price discovery efficiency and has the best performance.

In terms of the long memory of absolute returns, the Hurst exponent under VWAP strategy is the highest (0.4749), which is closest to the scenario of no-liquidation, followed by equal-order strategy (0.4589) and IS strategy (0.4563). By statistical analysis, the *p*-value of the significance test of the Hurst exponent under the equal-order strategy and the IS strategy is 0.3629, indicating that there is no significant difference between them. The Hurst exponent under VWAP strategy is significantly different from the equal-order strategy and IS strategy (both p-values are 0.0000), indicating that the VWAP strategy has the least impact on the market long memory of absolute returns.

5 Conclusions

To better understand the effects of the liquidation of large positions in the short term by institutional investors on the market, we design and establish a simulated market based on the multi-agent model method, including fundamentalists, chartists and random traders, to simulate the actual stock trading market. The simulation results can reproduce the stylized facts in the real financial market well, indicating that our model provides a suitable experimental platform for the subsequent analysis.

Then, we introduce one institutional trader into the basic model of the simulated market and study the liquidation performance of the equal-order, VWAP and IS strategies respectively. Finally, we compare and analyze the effects of the liquidation behavior under these three strategies on market liquidity, volatility, price discovery efficiency and long memory of absolute returns under the same market environment settings, and conclude the following: (1) compared with the equal-order strategy and VWAP strategy, the IS strategy can significantly reduce the liquidation cost, but causes the largest standard deviation of the liquidation cost and the longest liquidation time; (2) by comparing the data with and without liquidation behavior, we find that the institutional liquidation will reduce the market liquidity, price discovery efficiency and long memory of absolute returns, and increase the market volatility; (3) through the horizontal comparison of the three liquidation strategies, it is found that the IS strategy has the best performance in reducing the impact of liquidation behavior on the market liquidity, volatility and price discovery efficiency, while VWAP strategy has the least impact on the long memory of absolute returns.

Currently, this paper assumes that there is only one risk asset in the market, and only discusses the effects of liquidating large positions on the market within the range of several commonly used algorithmic trading strategies based on single asset, without considering the scenarios of multiple assets and corresponding more complicated liquidation strategies for portfolios. These will be further considerations in future research.

Acknowledgements This manuscript is supported by the National Natural Science Foundation of China (Grant No. 71671017).

Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix 1: Fat-tail Index (Shape parameter)

A large number of empirical studies show that the logarithmic return distribution has an obvious fat tail characteristic, and the probability of extreme changes is significantly higher than the corresponding probability of the normal distribution. The tail of the normal distribution decays exponentially to zero, while the tail of the logarithmic returns approximately decays as a power function, namely, high kurtosis. We use the generalized error distribution (GED) to measure the leptokurtic and fat tail characteristic of logarithmic returns of financial assets. The probability density function of GED is as follows:

$$f_s(x) = \frac{\lambda s}{2\Gamma\left(\frac{1}{s}\right)} exp\{-|\lambda(x-\mu)|^s\} \tag{6.1}$$

where $\lambda = \left[\frac{\Gamma\left(\frac{3}{s}\right)}{\Gamma\left(\frac{1}{s}\right)} \right]^{\frac{1}{2}}$, and s is the shape parameter. When $s=2$, GED degenerates to a normal distribution; when $s = 1$, GED degenerates to a Laplace distribution. When $0 < s < 2$, we can consider that the distribution has the characteristic of fat tail and call the shape parameter s the fat-tail index. The closer the value of the fat-tail index is to 2, the less significant the distribution's fat-tailed characteristic is; otherwise, the closer it is to 0, the more significant the fat-tailed characteristic is. The measurement of s can be obtained by solving the maximum likelihood function. The specific process is as follows.

According to formula (6.1), the maximum likelihood function of shape parameter s of GED can be obtained. Assuming N observation values, then after the zero-mean, we can obtain:

$$L(\lambda, s) = \prod_{i=1}^{N} f\left(x_i;\lambda, s\right) = \prod_{i=1}^{N} \frac{\lambda s}{2\Gamma\left(\frac{1}{s}\right)} exp\left\{-\left|\lambda x_i\right|^s\right\} = \frac{(\lambda s)^N}{\left(2\Gamma\left(\frac{1}{s}\right)\right)^N} exp\left\{-\sum_{i=1}^{N}\left|\lambda x_i\right|^s\right\} \tag{6.2}$$

$$\ln\left(L(\lambda, s)\right) = N\ln(\lambda) + N\ln\left(\frac{s}{2\Gamma\left(\frac{1}{s}\right)}\right) - \sum_{i=1}^{N}\left|\lambda x_i\right|^s. \tag{6.3}$$

According to the first order condition theorem, take the partial derivatives and set them equal to 0:

$$\frac{\partial ln(L(\lambda, s))}{\partial \lambda} = 0$$

$$\frac{\partial ln(L(\lambda, s))}{\partial s} = 0$$

thus, we have

$$\lambda = \left(\frac{N}{s\sum_{i=1}^{N}\left|x_i\right|^s}\right) \tag{6.4}$$

$$s + \Psi\left(\frac{1}{s}\right) + \ln\left(\frac{s}{N}\sum_{i=1}^{N}\left|x_i\right|^s\right) - \frac{s\sum_{i=1}^{N}\left|x_i\right|^s\ln\left|x_i\right|}{\sum_{i=1}^{N}\left|x_i\right|^s} = 0 \tag{6.5}$$

where $\Psi(t) = \frac{d}{dt}[\ln\Gamma(t)] = \int_0^1 \frac{1-x^{(t-1)}}{1-x}dx - \gamma$ and γ is Euler's constant. The tail shape parameters can be obtained according to Eq. (6.5).

Appendix 2

See Tables 7 and 8.

Table 7 Descriptive statistical results of the stock returns of 29 constituent stocks in SSE 50 in 2015

Stock	Mean	Std	Kurtosis	Skewness	Fat-tail index
SH000963	8.91E−06	1.00E−03	18.0100	−0.0635	0.8288
SH600011	1.07E−05	2.49E−03	24.8873	0.2124	0.2237
SH600018	5.41E−06	2.52E−03	21.4907	0.4547	0.0019
SH600019	8.20E−06	2.36E−03	16.9662	0.4324	0.1669
SH600021	2.08E−05	3.21E−03	37.3306	0.4469	0.4520
SH600048	7.94E−06	2.37E−03	14.3242	0.5071	0.2746
SH600050	2.17E−05	2.78E−03	19.0419	0.7197	0.2527
SH600068	9.36E−07	2.45E−03	24.9366	0.3426	0.1018
SH600309	4.41E−06	2.23E−03	60.9618	−0.4699	0.4006
SH600406	4.41E−06	2.74E−03	17.0823	0.2994	0.3605
SH600549	3.61E−06	2.53E−03	19.5371	0.5369	0.4319
SH600583	1.10E−05	2.65E−03	27.8334	0.3642	0.2714
SH600660	4.73E−06	1.78E−03	19.8310	0.1195	0.2543
SH601006	5.34E−06	2.37E−03	26.5444	0.2716	0.0024
SH601328	5.30E−06	2.19E−03	33.5682	0.4988	0.0028
SH601628	1.06E−05	2.29E−03	27.5759	0.7034	0.6756
SH601857	1.32E−05	2.10E−03	24.6780	0.6055	0.2747
SH601958	9.43E−06	2.70E−03	16.0002	0.2162	0.2711
SH601998	1.24E−05	2.50E−03	20.4184	0.5204	0.2142
SZ000001	3.14E−06	1.83E−03	31.4390	0.0402	0.1970
SZ000100	4.31E−06	2.53E−03	24.5485	−0.1674	0.1095
SZ000333	2.75E−06	2.03E−03	26.9421	0.2254	0.5101
SZ000338	−1.19E−06	2.06E−03	23.1864	0.1128	0.1847
SZ000402	3.25E−06	2.50E−03	14.2979	0.2119	0.0030
SZ000538	1.12E−05	1.83E−03	43.7585	0.8228	0.5035
SZ000651	4.95E−06	2.01E−03	32.4358	0.1967	0.5083
SZ000895	3.99E−06	1.71E−03	33.5028	0.0490	0.3844
SZ002475	1.47E−05	2.91E−03	36.9480	0.6503	0.4428
SZ300003	1.07E−05	3.27E−03	55.0436	−0.1239	0.3990

Table 8 Descriptive statistics of 1000 simulation returns (10 days, 30 days, 40 days, 50 days)

		Mean	Std	Kurtosis	Skewness	Fat-tail index	Hurst exponent
10 days	Mean	9.09E−07	3.52E−03	22.6803	−0.0142	0.7722	0.6575
	Std	3.03E−05	3.20E−04	12.7423	0.3652	0.0448	0.0553
	Max	1.05E−04	4.81E−03	147.5525	1.4370	0.9225	0.8257
	Min	−9.31E−05	2.56E−03	6.7174	−1.6871	0.6434	0.4956
	Median	1.49E−06	3.51E−03	19.2337	0.0010	0.7708	0.6563
30 days	Mean	−4.85E−07	3.52E−03	24.4289	−0.0014	0.7714	0.6234
	Std	1.73E−05	2.23E−04	9.7234	0.2467	0.0290	0.0506
	Max	5.79E−05	4.24E−03	153.5047	0.7913	0.8782	0.7937
	Min	−7.08E−05	2.85E−03	11.2242	−0.8374	0.6556	0.4878
	Median	−3.48E−07	3.52E−03	22.0190	−0.0027	0.7716	0.6197
40 days	Mean	−5.65E−07	3.53E−03	25.0229	−0.0016	0.7751	0.6012
	Std	1.50E−05	2.19E−04	7.9275	0.2239	0.0263	0.0499
	Max	4.57E−05	4.34E−03	74.8680	0.9488	0.8689	0.7369
	Min	−4.39E−05	2.87E−03	12.0905	−0.6308	0.6556	0.4489
	Median	5.00E−08	3.52E−03	23.4000	−0.0056	0.7758	0.5996
50 days	Mean	−3.96E−07	3.53E−03	25.3672	0.0203	0.7790	0.5911
	Std	1.41E−05	2.23E−04	7.9401	0.2066	0.0252	0.0510
	Max	4.45E−05	4.51E−03	102.1795	0.7827	0.8497	0.7814
	Min	−4.54E−05	2.88E−03	12.0835	−0.9501	0.6765	0.4500
	Median	−3.05E−07	3.52E−03	23.9569	0.0180	0.7798	0.5905

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