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ORIGINAL PAPER

Competition of spiral waves in heterogeneous CGLE systems

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Abstract The effects of spatial heterogeneity on a two-dimensional complex Ginzburg–Landau equation model are studied. In general, the interaction of a pair of spiral waves with a large degree of heterogeneity in two different media will cause three different patterns: (a) Multiple spiral waves coexist in different media; (b) the spiral wave is swept away in one medium and remains in another medium; (c) all of the spiral waves are suppressed by travelling waves having different frequencies. These travelling waves are generated from interface reported before. It is found that the interface is a wave source that can generate travelling waves with different frequencies in two submedia to compete with the original spiral waves in two different media. The competition results depend on the frequencies of the original spiral wave and the two travelling waves. Furthermore, local periodic pacing can replace the effect of the interface and reproduce the corresponding results, which gives additional evidence that the interface works as a wave source. The results give new ideas in pattern control such that we can suppress and annihilate spiral waves by generating a large degree of heterogeneity using selected parameters.

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1 Introduction

Spiral waves have been widely observed in natural systems, including biological systems, physical systems, and chemical reactions. Examples include cardiac muscle tissue [1–3], aggregating slime mold [4], oxidation of CO on platinum [5], and the Belousov–Zhabotinsky (BZ) reaction [6]. Because of its close relevance to cardiac arrhythmias, especially ventricular fibrillation (VF), which can cause sudden cardiac death in only 1 min, spiral waves are currently attracting much attention. An increasing number of experiments indicate that VF is induced by the transitions of cardiac electric propagating waves from spiral waves to turbulent waves [7]. Therefore, it has great significance to develop effective methods to control and annihilate spiral waves. There are various control approaches, including external signals [8–15], feedback control methods [16], parameter modifications [17], and heterogeneous patching method [18–20]. In this paper, we consider the effect of heterogeneity on spiral wave control.

The rules of spiral wave competition have been investigated and classified. In a homogeneous oscillatory medium, among outwardly propagating spirals (OPSs), higher frequencies dominate; among inwardly propagating spirals (IPSs), lower frequencies dominate; between OPSs and IPSs, OPSs dominates [18, 19,21,22]. However, the real systems are often spatially heterogeneous, so it is more complicated. Many studies of spiral waves in excitable and oscillatory systems have focused on the effect of heterogeneity, which have been extensively theoretically [18,23,24] and experimentally examined. For the competition of OPSs or IPSs, a weak heterogeneity causes the collision and annihilation of spiral waves with different frequencies [25,26]. The weak inhomogeneity causes certain spiral waves that are favorably located in the inhomogeneous medium to widen their domains, which crowd out and sweep away less favorably located spiral waves [21]. When the degree of heterogeneity is beyond the critical threshold, two IPSs coexist insulated by regions of highly disordered wave breakup in the midline region protected the core of the spiral wave from being unwound and swept away [18]; two OPSs definitely coexist because the induced wave front with the larger frequency cannot expand in the new territory [26]. For the competition of OPSs with IPSs, interface-selected waves (ISWs) were found in bidomain systems when the control parameters are on a well-defined parameter surface [19]. The aforementioned results were verified by researchers in the simulations or real experiments. However, when the degree of heterogeneity is sufficiently strong, multiple spiral waves do not always coexist, and we find other new patterns. In this article, we investigate the interaction of spiral waves with a large degree of heterogeneity; we find that not only two OPSs or two IPSs can coexist, but also that OPS can coexist with IPS and find a new different coexist mechanism. This mechanism can also illustrate the other new patterns. With this type of heterogeneity, the interface acts similar to a wall that block two media. It enables two types of waves with different frequencies in two media to coexist. We find that travelling waves generate from the interface, and they have different frequencies in these two submedia. Newly generated travelling waves have competitions for space with original spiral waves. Competitions between these waves are complex and some waves survive. If all spiral waves win in two media, multiple spiral waves coexist. However, when the generated travelling waves dominate, other patterns will appear, and the number of spiral waves will decrease. Our research investigates the type of heterogeneity that can reduce or eliminate spiral waves.

In this paper, with the aforementioned motivations, the interactions between stable spiral waves in a two-dimensional heterogeneous oscillatory medium are investigated in detail. Using the two-dimensional complex Ginzburg–Landau equation (CGLE), the formation mechanisms of the travelling waves that originate from the competition among spiral waves in different media are explored. In the next section, we recall the two-dimensional CGLE model. In Sect. 3, we present the numerical simulations and analysis for the competition of spiral waves in heterogeneous media. We find that a local periodic pacing can replace the effect of the interface and reproduce all corresponding results, which demonstrates that the interface is a wave source. Conclusions and discussions are provided in Sect. 4.

2 Model

A two-dimensional complex Ginzburg–Landau equation (CGLE) is investigated. It describes extended systems near a Hopf bifurcation and where the homogeneous state is oscillatory. The CGLE system is as follows:

$$\frac{\partial W}{\partial t} = W - (1 + i\alpha)|W|^2 W + (1 + i\beta)\nabla^2 W \tag{1}$$

where $W(x, y, t)$ is a complex variable that describes the amplitude of the pattern modulations at spatial location (x, y) and time t; α and β are the nonlinear frequency shift and diffusion coefficient, respectively. ∇^2 is the two-dimensional Laplacian operator. To study the interaction of two spiral waves in different media, a simple model is constructed with two halves of the medium having different α or β.

$$(\alpha, \beta) = \begin{cases} (\alpha_L, \beta_L), & x \in [1, 200], \quad y \in [1, 200], \\ (\alpha_R, \beta_R), & x \in [201, 400], \quad y \in [1, 200]. \end{cases} \tag{2}$$

In the simulations, the CGLE system is integrated by an explicit Euler method with time step $\Delta t = 0.01$, and the standard five-point approximation is selected to discretize the Laplace operator with the space step $\Delta x = \Delta y = 0.5$. Each half of the medium is a 200×200 grid with no-flux boundary conditions. The spiral waves in each homogeneous medium are generated by the cross-field initial condition in all simulations unless otherwise specified.

Because the results of the interaction of two spiral waves crucially depend on the frequency of the spiral

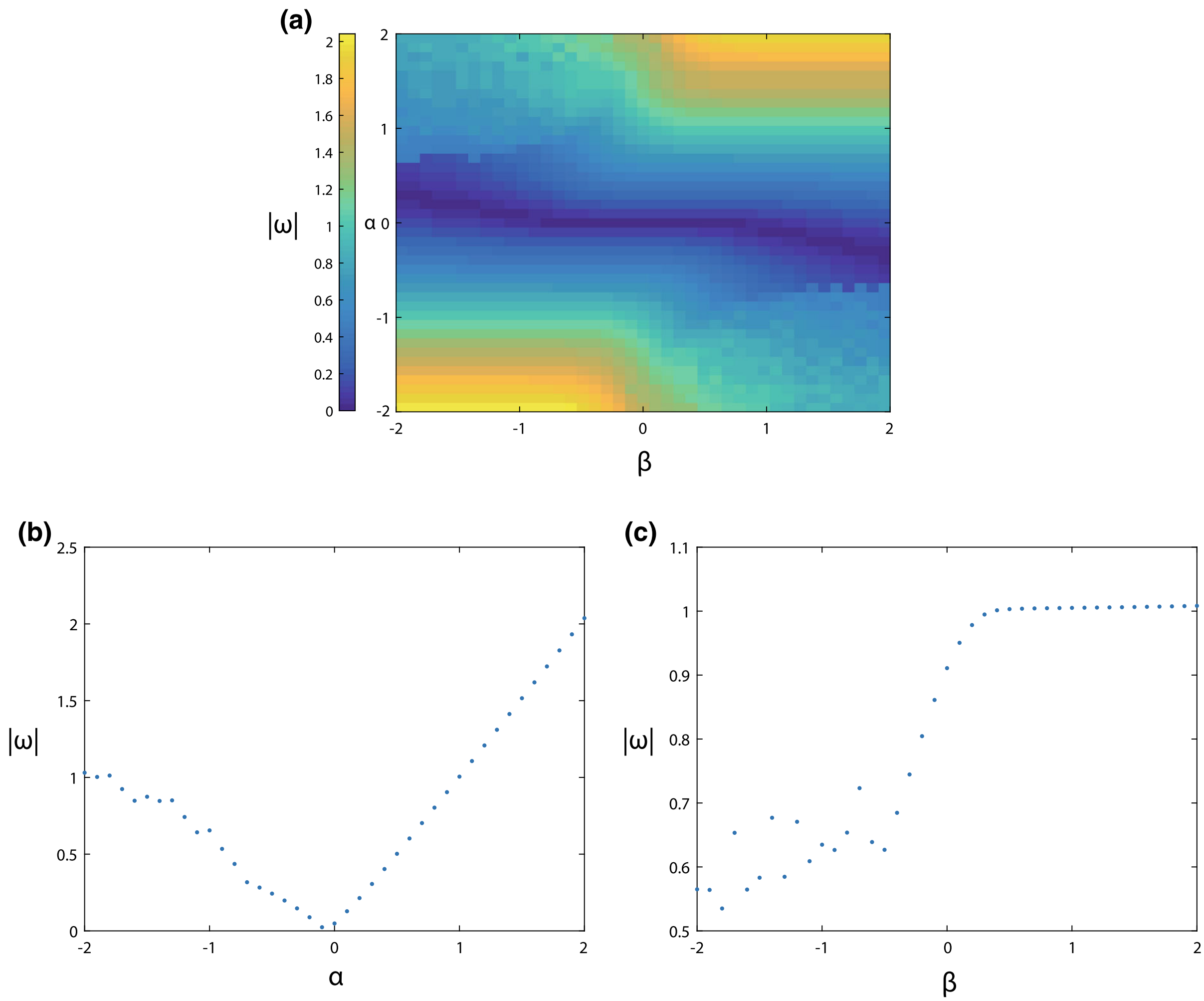


Fig. 1 (Color online) **a** The frequency (ω) of a single spiral wave in parameter space (α, β) of the CGLE system. **b** The frequency of a single spiral wave with $\beta = 1$. **c** The frequency of a single spiral wave with $\alpha = 1$

waves, we have numerically obtained the corresponding frequency on the parameter space. Figure 1a shows the frequency of a single spiral wave in the CGLE system. Figure 1b shows the frequency of a single spiral wave with $\beta = 1$. Figure 1c shows the frequency of a single spiral wave with $\alpha = 1$.

3 Numerical results and analysis

3.1 Competition of spiral waves

The competition of spiral waves in heterogeneous CGLE systems is studied. For simplicity, we first generate two stable spirals and subsequently investigate the interaction between two spiral waves in the left and right halves of the medium. All results are stable patterns after sufficient steps of simulation. There are three different patterns after the interaction: coexistence of multiple spiral waves with different parameters, one remaining spiral wave and no remaining spiral wave [see supplementary]. All results are illustrated in detail in the following sections.

3.1.1 Coexistence of multiple spiral waves with different parameters

Figure 2 shows the results of two remaining spiral waves, where the red arrows indicate the propagation direction of the waves. Figure 2a–c shows the

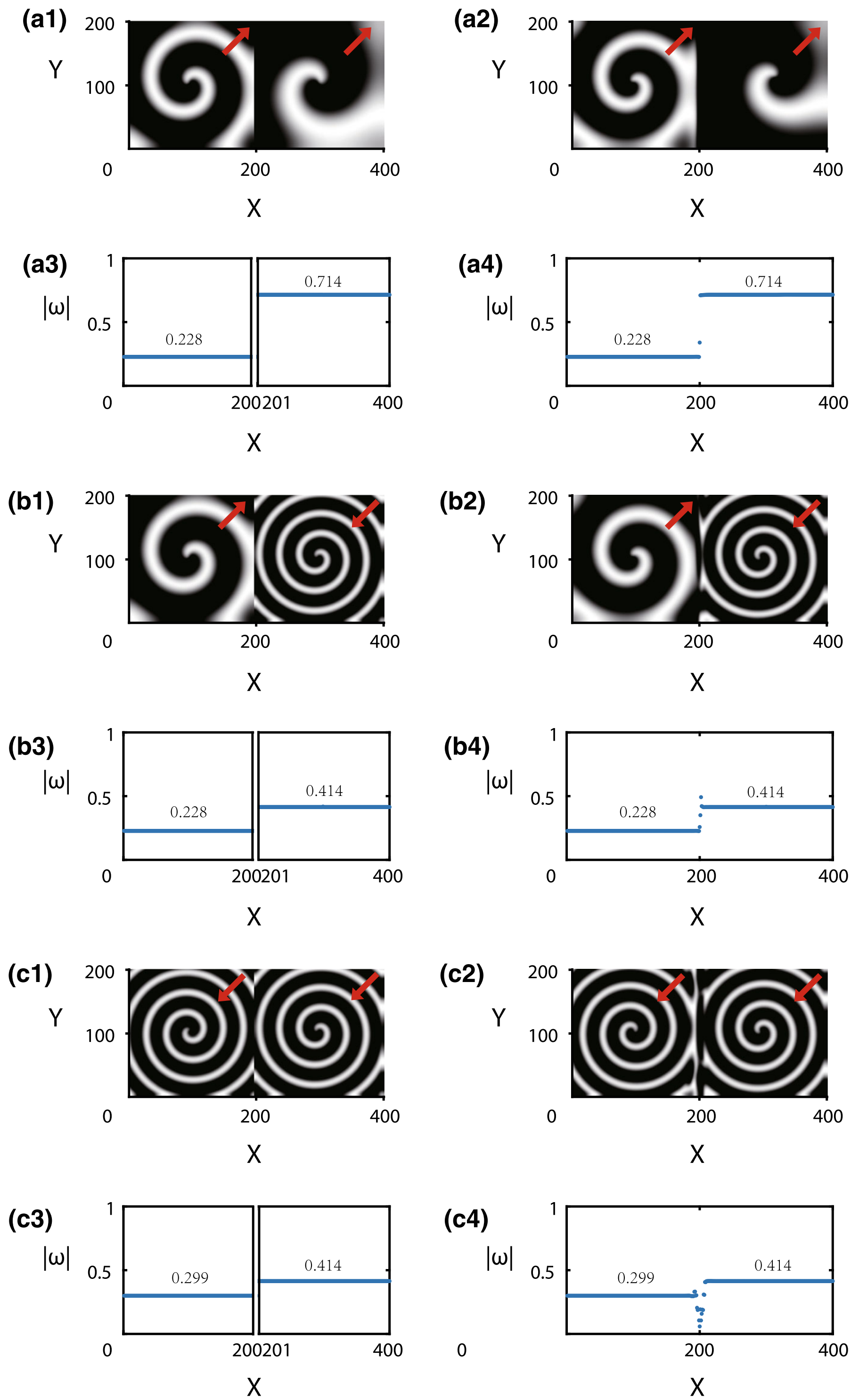


Fig. 2 (Color online) Spiral waves competition patterns of $W(x, y)$'s real part, of a heterogeneous 2D CGLE system with coexistence of multiple spiral waves after the competition. The red arrows indicate the propagation direction of the waves, the blue dots show the numerical results of frequency distributions at $y = 100$, $x \in [1, 400]$. All the corresponding frequencies of the steady states are labeled in the figures. Each half of the medium is a 200×200 grid with no-flux boundary conditions. Numerical simulations are made with the space step $\Delta x = \Delta y = 0.5$, time step $\Delta t = 0.01$. The spiral waves in each homogeneous medium are generated by the cross-field initial condition and the above time and space steps, and initial condition are used in all simulations unless otherwise specified. **a** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = -0.7$, $\beta_R = -2.2$. **b** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = -0.6$, $\beta_R = 0.6$. **c** $\alpha_L = 0.5$, $\beta_L = -0.8$, $\alpha_R = -0.6$, $\beta_R = 0.6$. **a1–c1** and **a3–c3** are the steady states and steady frequencies before the competition of spiral waves. **a2–c2** and **a4–c4** are the steady states and steady frequencies after the competition of spiral waves. After the competition, there have multiple spiral waves coexist, the difference in the frequency of the spiral waves in the two media also indicates that multiple spiral waves can coexist

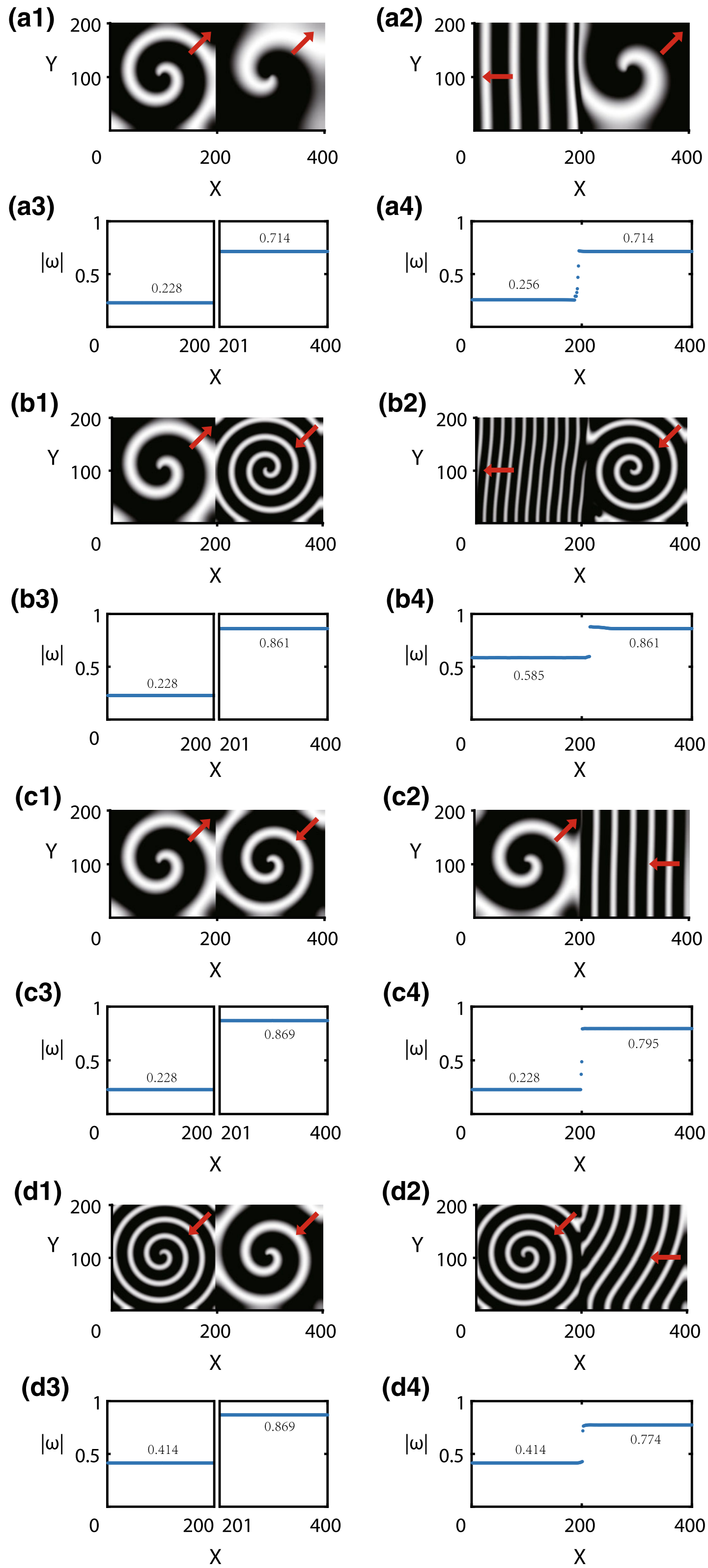


Fig. 3 (Color online) One spiral wave remnant. **a** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = 0.7$, $\beta_R = 2.2$. **b** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = 1.0$, $\beta_R = -0.1$. **c** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = -0.9$, $\beta_R = -0.1$. **d** $\alpha_L = -0.6$, $\beta_L = 0.6$, $\alpha_R = -0.9$, $\beta_R = -0.1$. **a1–d1** and **a3–d3** are the steady states and steady frequencies before the competition of spiral waves. **a2–d2** and **a4–d4** are the steady states and steady frequencies after the competition of spiral waves. After the competition, the spiral wave in one domain is driven away by the generated travelling waves, the difference in the frequency of the spiral wave, and the travelling waves indicates they can coexist

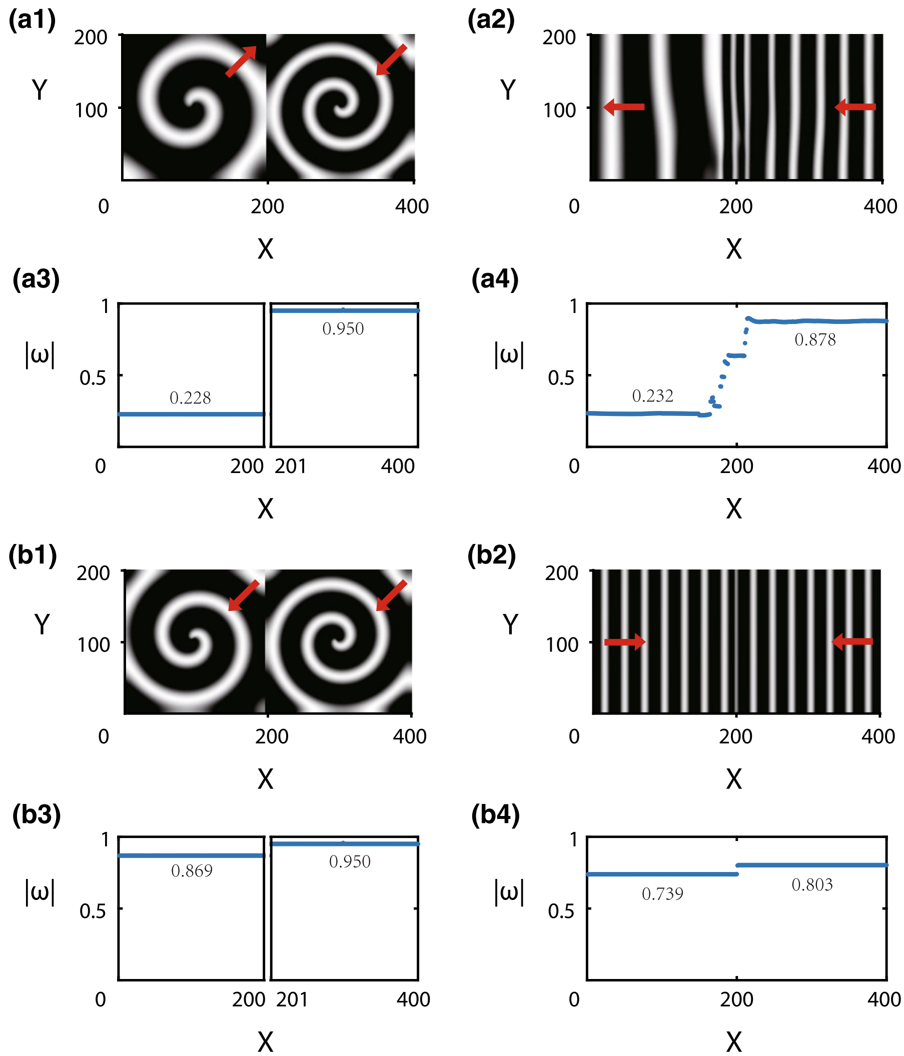


Fig. 4 (Color online) No spiral wave remnant. **a** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = 1.0$, $\beta_R = 0.1$. **b** $\alpha_L = -0.9$, $\beta_L = -0.1$, $\alpha_R = 1.0$, $\beta_R = 0.1$. **a1**–**b1** and **a3**–**b3** are the steady states and steady frequencies before the competition of spiral waves. **a2**–**b2** and **a4**–**b4** are the steady states and steady frequencies after the competition of spiral waves. After the competition, the spiral waves in the two domains all become travelling waves, the difference in the frequency of the travelling waves indicates the generated travellings can coexist

cases of competition with two OPSs, one OPS and one IPS, and two IPSs, respectively. The corresponding parameters are provided in the figure caption. Figure 2a1–c1 shows the steady states before the competition. Figure 2a2–c2 shows the steady states after the competition. Figure 2a3, a4 (Fig. 2b3, b4, c3, c4) shows the numerical results of frequency distributions at $y = 100$, $x \in [1, 400]$. All corresponding frequencies of the steady states are labeled in the figures. We observe that after the interaction, these spiral waves in two domains coexist, and the frequencies remain unchanged. These results are consistent with previous results that when the heterogeneity becomes sufficiently large, multiple spiral waves can coexist [18].

3.1.2 One spiral wave remnant

Figure 3 shows the results that only one spiral wave remains after the competition. The other spiral wave was swept out of the boundary by the travelling waves. In Fig. 3a, there are two OPSs; after the interaction, the left OPS was suppressed by the travelling waves, and the right OPS remains. Clearly, the travelling waves have a larger frequency than the original spiral wave. In Fig. 3b, c, there are one OPS in the left medium and one IPS in the right medium. The left OPS becomes travelling waves in Fig. 3b2, and the right IPS becomes travelling waves in Fig. 3c2. In Fig. 3d, there are two IPSs; after the interaction, the right spiral wave becomes small-frequency travelling waves. It is interesting to compare these frequencies before and after the competition. We find that when the travelling waves suppress the OPS, the frequency will increase. When the travelling waves suppress the IPS, the frequency will decrease. The frequency of the remnant spiral wave remains unchanged.

3.1.3 No spiral wave remnant

Figure 4 shows the results that there is no spiral wave after the competition. In Fig. 4a, there is one OPS and one IPS. After the competition, both domains become travelling waves. The frequency of the left domain increases, and the frequency of the right domain decreases. There are two IPSs in Fig. 4b. After the interaction, the frequencies decrease in both domains. For the two OPSs, we have attempted many cases but unfortunately have not found an instance where both OPSs become higher-frequency travelling waves after the competition.

3.2 Generated travelling waves and their effects on the original spiral waves

According to the above results, when the heterogeneity between the left and right media is sufficiently strong, the interface becomes a wave source and generates travelling waves. The travelling waves have different frequencies in the two different media, which is not similar to the interface-selected waves with same frequency in submedium at all. In every medium, the original spiral waves compete with the generated travelling waves obeying the competition laws in homogeneous medium. When the original spiral wave is an OPS, the high-frequency wave suppresses the low-frequency wave. When the original spiral wave is an IPS, the low-frequency wave suppresses the high-frequency wave. Thus, the generated travelling waves have crucial roles in the competition of two spiral waves.

The above facts can be well explained by the frequency competition of the generated travelling waves from the interface with the corresponding stable spiral waves in the same medium. Previous studies have claimed that the interface is a wave source generating travelling waves with the same frequency in the submedium in some conditions [19,27]. To further justify these conclusions, we alter the media by gradually changing the parameter setting. The following simulation results can provide us more evidence about these conclusions.

We fixed $\alpha_L = 0.2$ and $\beta_L = 1.2$ (point A in Fig. 5a) in the left medium, set $\alpha_R = 1.0$ (l_1) and $\alpha_R = -0.9$ (l_2) and altered $\beta_R \in [-0.5, 0.5]$ in the right medium. Figure 5b, c shows the frequencies after the interaction between A and l_1 (l_2). The black-dashed curves represent the frequencies of the original spiral waves before the competition (ω_L, ω_R), and the black dotted lines represent the frequencies of the generated travelling waves after the competition (ω'_L, ω'_R). To obtain the frequencies of the generated travelling waves, we first set the travelling waves in the left and right media and subsequently let the two media interact with each other. The frequencies are measured at the left ($x = 50$, $y = 100$) and right ($x = 350$, $y = 100$) sides of the medium after the system reaches the steady state. Because the left medium remains unchanged, the frequency of the original spiral wave (ω_L) is always constant. Meanwhile, when β_R is changed, ω_R, ω'_L and ω'_R are altered. Thus, with different parameters, there are different patterns after the competition. The points linked by the blue lines (AB, AD) and green line (AC) represent the corresponding results in Figs. 3b, c and 4a. The points linked by the red line (AE) represents the competition between A and E. There are one OPS in the left medium and one IPS in the right medium. In the left medium, ω_L is larger than ω'_L; in the right medium, ω_R is smaller than ω'_R. According to our conclusions, two spiral waves should coexist. Figure 6 depicts the steady states and steady frequencies before and after the competition. It shows that the prediction result actually occurs in the simulation.

Now, we attempt to show that the interface can act as a wave source. In other words, a local periodic pacing as the wave source can generate the above travelling waves in two media. For the system:

$$\begin{cases} \frac{\partial W}{\partial t} = W - (1 + i\alpha_L)|W|^2 W + (1 + i\beta_L)\nabla^2 W, & x \in [1, 200], \quad y \in [1, 200], \\ W = I, & x = 201, \quad y \in [1, 200], \\ \frac{\partial W}{\partial t} = W - (1 + i\alpha_R)|W|^2 W + (1 + i\beta_R)\nabla^2 W, & x \in [202, 400], \quad y \in [1, 200]. \end{cases} \tag{3}$$

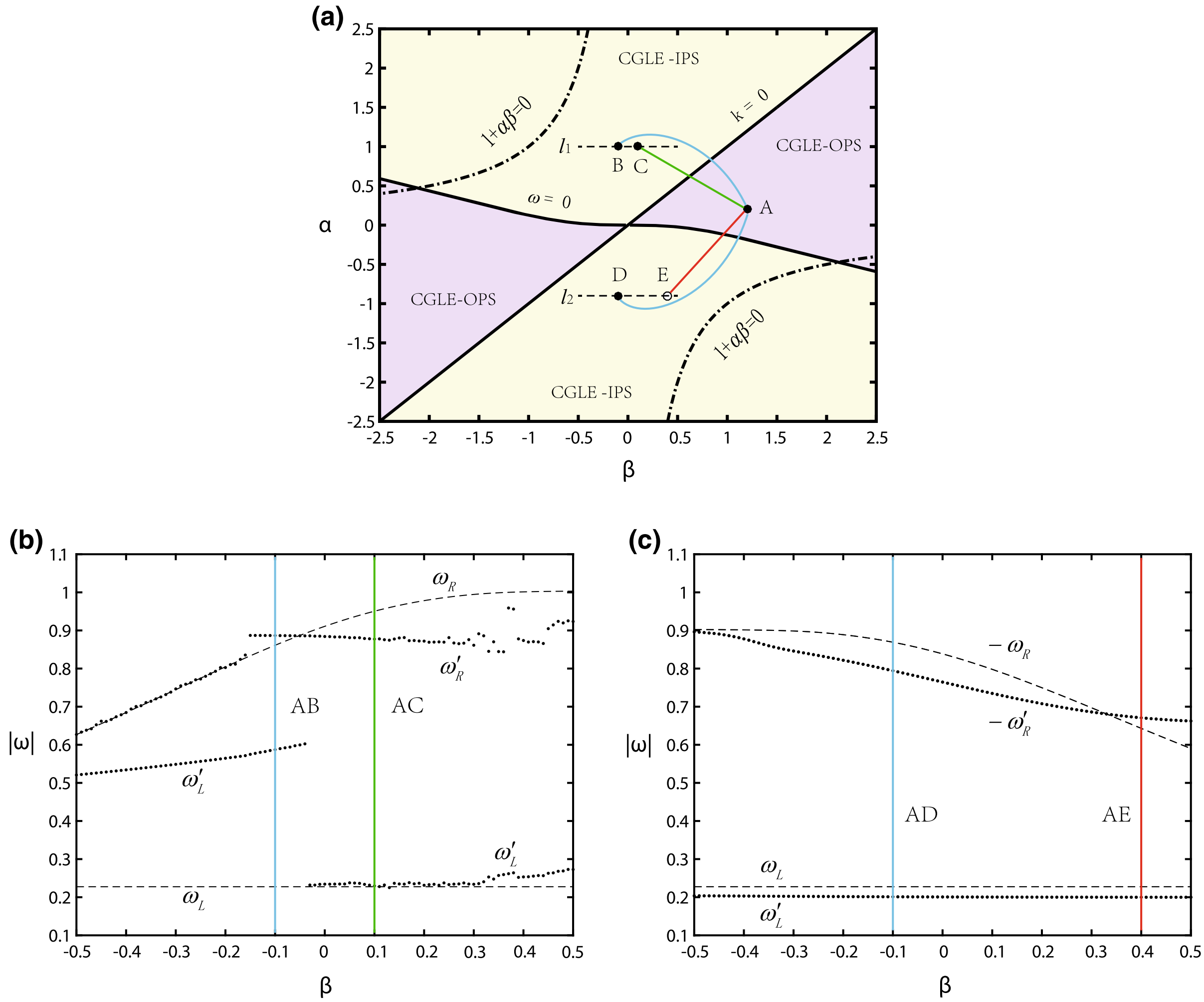


Fig. 5 (Color online) **a** Parameter space (α, β) of the CGLE. A: $\alpha = 0.2$, $\beta = 1.2$, B: $\alpha = 1.0$, $\beta = -0.1$, C: $\alpha = 1.0$, $\beta = 0.1$, D: $\alpha = -0.9$, $\beta = -0.1$, E: $\alpha = -0.9$, $\beta = 0.4$. The points linked by red lines represent coexistence of multiple spiral waves with different parameters (see Fig. 2), the points linked by blue lines represent one spiral wave remnant (see Fig. 3), and the points linked by green lines represent no spiral wave remnant (see Fig. 4). **b** Frequency competition between the original spiral waves and the generated travelling waves in each half medium. The frequencies are measured at the left ($x = 50$, $y = 100$) and right ($x = 350$, $y = 100$) sides of the medium after the system reaches the steady state. Left medium: A, right medium: $\alpha_R = 1.0$, $\beta_R = \beta$ (l_1). The black-dashed curves represent the frequencies of the original spiral waves before the competition (ω_L, ω_R), and the black dotted lines represent the frequencies of the generated travelling waves after the competition (ω'_L, ω'_R) with a initial condition that generate travelling waves. **c** The same as (**b**) with left medium: A, right medium: $\alpha_R = -0.9$, $\beta_R = \beta$ (l_2)

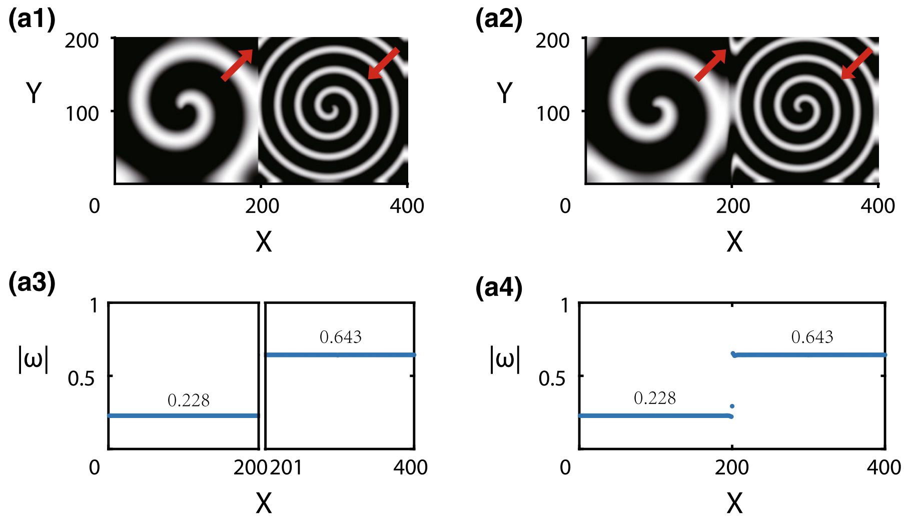
It supports a travelling wave solution

$$W = \sqrt{1 - k^2} e^{i(-\omega t + kr)}, \tag{4a}$$

$$\omega = \alpha + (\beta - \alpha) k^2. \tag{4b}$$

where I is the local periodic pacing; k is the wavenumber; and $r = \sqrt{x^2 + y^2}$. In a single medium, when there is no local periodic pacing, ω is the natural frequency ω_0 ($\omega_0 = \alpha$ for CGLE). When we let the local periodic pacing I be a wave source in the $x = 201$ stripe at the interface of the two different media, the travelling waves will be induced in the left and right media by the pacing. With the pacing, we know that the frequencies of the generated travelling waves in two coupled media

Fig. 6 (Color online) The spiral waves' competition between A and E. **a1** and **a3** are the steady states and steady frequencies before the competition. **a2** and **a4** are the steady states and steady frequencies after the competition



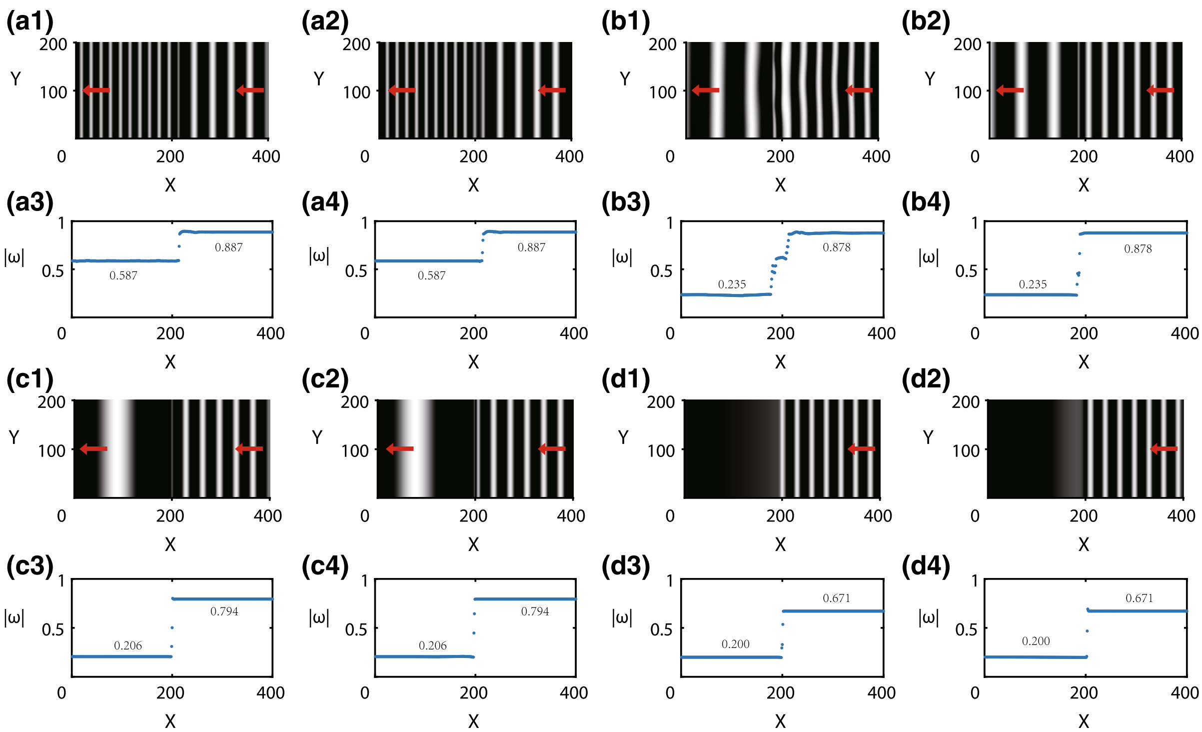


Fig. 7 (Color online) Compare the influence of the interface and the periodic pacing. We gave an initial condition that can generate travelling waves in two different media and then let them compete with each other. **a1–d1** and **a3–d3** are the steady states and steady frequencies after the competition of the initial travelling waves with the generated travelling waves. We applied a local periodic pacing I as the wave source to the $x = 201$ stripe at the interface of the two different media. **a2–d2** and **a4–d4** are the steady states and steady frequencies after the competition of the initial travelling waves with the travelling waves that induced by the pacing. **a** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = 1.0$, $\beta_R = -0.1$, $\omega'_L = 0.587$, $\omega'_R = 0.887$; **b** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = 1.0$, $\beta_R = 0.1$, $\omega'_L = 0.235$, $\omega'_R = 0.878$; **c** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = -0.9$, $\beta_R = -0.1$, $\omega'_L = 0.206$, $\omega'_R = -0.794$; **d** $\alpha_L = 0.2$, $\beta_L = 1.2$, $\alpha_R = -0.9$, $\beta_R = 0.4$, $\omega'_L = 0.200$, $\omega'_R = -0.671$. The frequency distributions and wave numbers after the competition have the same characteristics as the frequency distributions and wave numbers after the pacing as a wave source added at the interface

are ω'_L and ω'_R. If we insert Eqs. (4b) into (4a) without the coupling terms, every grid in the left and right media is a rotator:

$$W_L = \sqrt{1 - \frac{\omega'_L - \alpha_L}{\beta_L - \alpha_L}} e^{i(-\omega'_L t)}, \tag{5a}$$

$$W_R = \sqrt{1 - \frac{\omega'_R - \alpha_R}{\beta_R - \alpha_R}} e^{i(-\omega'_R t)}. \tag{5b}$$

Thus, it is reasonable to let the local periodic pacing be $I = W_L + W_R$ and reproduce similar travelling waves in two media.

Figure 7a1–d1 shows the steady states after the interaction of travelling waves in two different media. Figure 7a2–d2 shows the steady states with the local periodic pacing at the $x = 201$ stripe. Figure 7a3–d3, a4–d4 shows the corresponding steady frequencies. In Fig. 7b–d, with the local periodic pacing $I = W_L + W_R$ as the wave source, we can generate the travelling waves with identical wavenumbers and frequencies in each medium as shown in two coupled media. However, this type of pacing cannot obtain a similar result in Fig. 7a. In Fig. 7a, $\omega'_L = 0.587$, where the system is completely driven by the pacing [see supplementary]. Thus, the system is sensitive to the pacing, and $I = W_L + W_R$ does not work for this case. We find that the pacing $I = 0.55e^{i(-\omega'_L t)}$ works well for the two regions including the gradual change in frequencies. These numerical results demonstrate that with a large degree of heterogeneity in two different media, the interface can act as a wave source.

4 Conclusion

Spiral wave competition has become an interesting question in recent years. In this study, we have investigated the competition of spiral waves in two different media with a large degree of heterogeneity using a model of a two-dimensional CGLE system.

We have found a new phenomenon which was not reported in literature before as per author's knowledge. In summary, there are three results for different parameters in this system: coexistence of multiple spiral waves with different parameters, one remaining spiral wave and no remaining spiral waves. We have also found that travelling waves can generate from the interface and propagate to two submedia when the degree of heterogeneity of both media is beyond their critical thresholds [18]. The interface acts similar to a wall that blocks two media and becomes a wave source. Then, the competition of spiral waves in these media becomes the competition of the generated travelling waves from the interface with the original spiral wave in each homogeneous medium. The competition results depend on the frequencies of the original spiral waves, and the generated travelling waves from the interface may have different frequencies in two media. The interface as the wave source plays a crucial role in the spiral wave competition. Detailed research on the interface as a wave source can deepen our understanding. Thus, we have proven that the interface is a wave source by adding a local periodic pacing at the interface, and an appropriate local periodic pacing can generate identical results.

The aforementioned features suggest that we can eliminate spiral waves by generating a large degree of heterogeneity using selected parameters. Although the above studies address the competition of spiral waves in heterogeneous CGLE systems, the conclusions are also applicable to more complex heterogeneous oscillatory media such as BZ-AOT systems. The proposed method may have potential applications in actual experiments.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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