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Quantifying financial market dynamics: Scaling law in rank mobility of Chinese stock prices

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A B S T R A C T

Rank mobility, which was designed to measure the average variation of relative rank positions with respect to any absolute variable over a given time period, can be used to explore how the memory of stock price ranking orders fades over time. We investigate the variations in rank order of the closing prices of stocks registered at the Shanghai A-share market over a long period of 16 years. And we find that rank mobility increases as a power law with increasing time scale, and eventually converges to a constant level. This power-law relationship can be observed not only over a long period of 16 years but also for each consecutive year, especially their power-law exponents are very close. The empirical evidence indicates a fundamental dynamics of Chinese stock price movements.

1. Introduction

Recent studies have shown that stock prices move together in the same market due to poor firm-specific return volatility (Jin and Myers 2006; Kelly 2014; David and Simonovska 2016). Such phenomenon is called stock price co-movement or synchronicity, and can be measured by the R^2 from asset pricing regressions (Morck et al., 2000). Stock price co-movement has been found in both the developed countries (see, for example, Cahan et al., 2009; Crawford et al., 2012; etc.) and the emerging economies (Khanna and Thomas 2009; Morck et al., 2000). According to the work of Morck et al. (2013), the fluctuation of stock price is attributed to two factors: one is the firm-specific information affecting few stocks; the other is market-wide information affecting most stocks. The firm-specific uncertainty is non-systematic risk that can be reduced by diversified investment (Beckman et al., 2004). On the other side, the market-wide information on stock prices leads to high stock price synchronicity and market inefficiency (Roll 1988). Compared with the impact of market-wide information on a stock market, those stocks with strong firm-specific fluctuation can change the ranking orders of their prices more likely. In this paper, we propose a measure of rank mobility to quantify the overall changes of price ranking orders of a certain number of stocks so that we can study the memory of stock price ranking orders.

Mobility is a term broadly used to refer to inequality of opportunity and dynamic inequality, in economics, education and politics, etc. (Morgan et al., 2006; Rodriguez et al., 2008). Rank mobility which belongs to the class of the relative mobility (Fields and Ok 1999), is defined as the average variation of relative rank positions with respect to any absolute variable over a given time period. Rank

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mobility measures have been widely applied in social dynamics, wealth redistribution and changes in income status (D'Agostino and Dardanoni 2009; Bossert et al., 2016). Rank mobility of stock prices reflects the changes of their price ranking orders which are mainly affected by firm-specific information. Since the market-wide information has impacts on most stocks, it will drive their prices to move in the same direction, thus has less direct impact on the ranking orders of stock prices.

It is important to note that the measurements of rank mobility strongly depend on the length of the interval between the two sampling time points. Huang and Wang (2014) demonstrated this time-dependence of rank mobility using both artificial data and survey panels of household income, and reported that the dependence function exhibits a specific profile that can be observed when data are sampled at sufficiently high rate. The dependence can also be established for the time series of financial markets, due to the high frequency of recordings of trading activity and stock prices (Wu et al., 2014).

In this study, we measure the extent of relative movement of stock prices with rank mobility, and we show that the dependence of rank mobility of stock closing prices recorded on the Shanghai A-share market on the time scale exhibits a stable scaling law over a broad range of time scales. Further, we find that this scaling law is present for different calendar years, indicating a robust behavior underlying the overall variations in the ranks of stock prices. These findings provide a new insight into understanding the global price dynamics of multiple stocks, especially in the aspect of stock price ranking changes during a certain time scale, and thus may infer the dynamics of the market from the changes in certain stock prices.

2. Data and methodology

The empirical data we utilize in this study is from the China Stock Market and Accounting Research Database (CSMAR), which spans from Jan. 1, 2001 to Dec. 30, 2016. Data on daily closing prices are adjusted for stock split, dividends and rights offerings, so that the stock price ranking will not change due to these events. Since in our analyses we trace rank variations for a fixed number of stocks during a long time period of 16 years, we drop the stocks that were delisted from the stock market within this period, and investigate the time series of $N = 503$ stocks over $T = 3876$ trading days.

We rank N stocks according to their daily closing prices with highest rank corresponding to the stock with highest closing price. The rank sequence of stocks at time t can be expressed as $\mathrm{R(t)} = (r_1(t), r_2(t), \ldots, r_N(t))$, where $r_i(t)$ denotes the rank of stock i at time t, and $i = 1, 2, \ldots, N$.

Rank mobility characterizes the difference between two rank sequences R(t) and R(t + Δt), where Δt is the time scale of observation. Given the rank time series of all stocks, the rank mobility between time points t and $t + \Delta t$ can be mathematically expressed as,

$$RM_{(t,\Delta t)} = \frac{1}{N^2}\sum_{k=1}^{N}|r_k(t) - r_k(t+\Delta t)|, \tag{1}$$

where N denotes the number of stocks we take into account, and $RM_{(t,\Delta t)}$ takes non-negative values, i.e., $RM_{(t,\Delta t)} \geq 0$. From the definition above, we can also find that RM actually represents the changes of stock price ranking order in the stock market during a given Δt at t. From a holistic perspective, it measures the extent to which the relative price changes inside a stock market. The smaller the RM value, the smaller the variation of stock price ranking order.

The most commonly used measurement of stock price synchronicity is R^2. It is used to capture the stock price synchronicity of a single stock, to be more specific, how the price of a single stock moves with respect to the market. Morck et al. (2000) argue that higher R^2 values reflect more market-wide information, and lower R^2 values reflect more firm-specific information. While Ashbaugh-Skaife et al. (2006) investigate the validity of the information-based interpretation of stock price synchronicity in six markets, and the results of their analyses suggest that the variation in stock price synchronicity (measured by R^2 value) across firms in international markets is not due to differences in firm-specific information. An alternative explanation for a high R^2 is that the fundamental drivers of firm value (market-wide information) are highly correlated. But that not necessarily means less firm-specific information.

Unlike R^2, RM is used to capture the average variation of price ranking orders of a certain number of stocks. Any information that affects price ranking orders of a subset of stocks will be reflected on the value of RM, whether it be market-wide information or firm-specific information. Generally, market-wide information affects the whole market, while firm-specific information affects a subset of stocks. Because of different betas and R^2 (from asset pricing regressions), different stocks have different reactions to the market. Therefore market-wide information may change the price ranking orders of stocks. RM also fills in the blank of a single indicator that has not yet portrayed the overall relative movement of stock prices. Furthermore, RM can also specify a change in extent of such movement with time interval. In contrast, R^2 is generated by regression on data which cannot be taken as a time variable. So RM is more convenient to describe the relationship between variation of stock price ranking orders and time interval. It provides a complementary approach to dynamic research of price movement.

According to the definition, $RM_{(t,\Delta t)} = 0$ when the two paired rank sequences $R(t)$ and $R(t + \Delta t)$ are exactly the same. As the difference between the two rank sequences increases, the value of $RM_{(t,\Delta t)}$ becomes larger, that is to say, the relative movement of stock prices increases. Under the assumption of independent and uniformly distributed ranks, the expectation value of $RM_{(t,\Delta t)}$ takes the following form,

$$E\left(RM_{(t,\Delta t)}\right) = \frac{1}{3} - \frac{1}{3N^2}. \tag{2}$$

When N is large enough, this expectation converges to 1/3. With increasing time interval Δt, the variation of the rank order in a rank sequence R(t) could be higher, and thus rank mobility $RM_{(t,\Delta t)}$ is expected to be larger. In our analysis, we measure the average rank mobility $\overline{RM}$ from Eq. (1) for a fixed time scale Δt and for consecutive days during a given time period of T. $\overline{RM}$ actually represents the average relative movement of stock prices in the market within Δt during a given time period of T.

We demonstrate that the averaged rank mobility follows a power law over a broad range of time scales $\Delta t \in [0, 1000]$ trading days. Our empirical analysis leads to the following functional relation,

$$\overline{RM} = \alpha \Delta t^{\beta}, \tag{3}$$

which is obtained by fitting the empirical data by Non-linear Least Square (short for NLS), and the parameters α and β are estimated using the Gauss–Newton method. When we present the above function on double logarithmic plot, the relation appears as a straight line, indicating a linear relation which takes the following form,

$$ln\overline{RM} = ln\alpha + \beta ln\Delta t. \tag{4}$$

3. Empirical Results

3.1. Average rank mobility for different time scales and calendar years

As the first step of our analysis, we calculate the average rank mobility $\overline{RM}$ for each calendar year in the time period from 2001 till 2016, given a fixed time scale Δt. We next repeat this calculation for the same calendar years but for different time scales Δt, and we trace the evolution of $\overline{RM}$ over the 16-year period (shown in Fig. 1). We find that, for each time scale Δt, $\overline{RM}$ varies slightly over consecutive calendar years. Moreover, the same mobility profiles for different Δt exhibit similar trends year by year, which can be also clearly observed by the rank mobility profile for one trading day in the inset of Fig. 1. Further, we find that the rank mobility of Chinese stock market goes up with increasing the time scale Δt for every year, indicating that the difference between two rank sequences becomes larger as Δt increases.

3.2. Scaling relation of average rank mobility

In this section, we examine the functional relation between rank mobility $\overline{RM}$ and time scale Δt. We find that $\overline{RM}$ exhibits a characteristic profile: a Regime I of rapid growth at small and intermediate time scales, followed by Regime II of saturation at large time scales (Fig. 2a). Log-log plot and theoretical fit of $\overline{RM}$ in Regime I reveal a scaling relation that remains stable over a broad range of time scales $\Delta t \in [0, 1000]$ trading days (Fig. 2b).

Table 1 shows the results of a non-linear fitting with Eq. (3) for the first 1000 data points of $\overline{RM}$, for consecutive time scales Δt. As shown in Fig. 2, the fitting curve overlaps the empirical one in Regime I and eventually departs after $\Delta t \geq 1000$. In order to clearly illustrate that the scaling behavior of $\overline{RM}$ in Regime I is in agreement with the theoretical fit, we plot both curves on a separate log-log

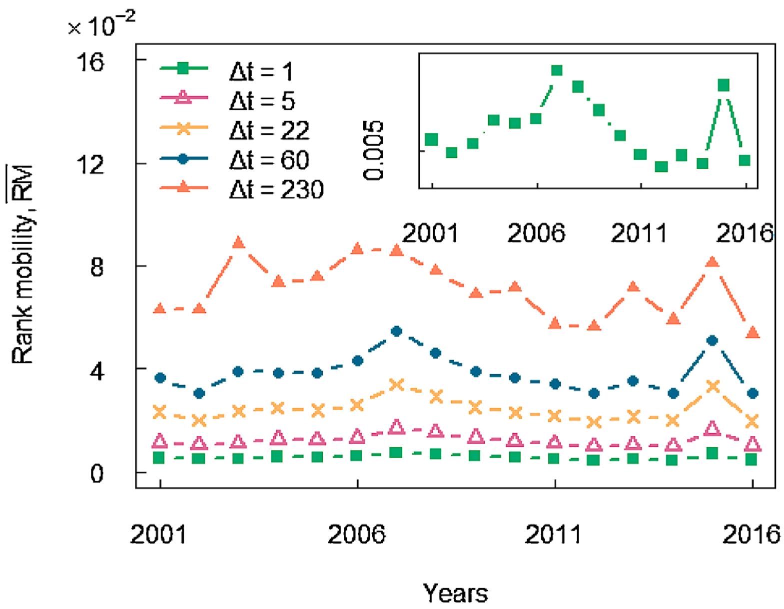

Fig. 1. Average rank mobility $\overline{RM}$ of daily closing stock prices for different time scales $\Delta t = 1, 5, 22, 60, 230$ trading days. The inset shows the case of $\Delta t = 1$ with higher resolution in the vertical scale to illustrate similar trends in $\overline{RM}$ across different Δt for the same data recording period of 16 years.

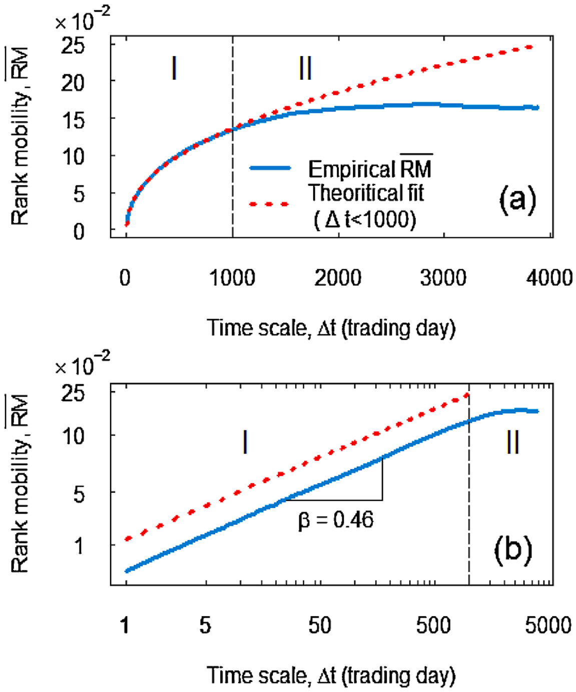

Fig. 2. Functional dependence of rank mobility $\overline{RM}$ on time scale Δt. (a) Linear-linear plot of $\overline{RM}$ versus Δt shows a Regime I of rapid growth on time scales $\Delta t \leq 1000$ trading days, followed by a saturation Regime II for time scales $\Delta t \geq 1000$. (b) Log-log plot of $\overline{RM}$ versus Δt in Regime I indicates a power-law scaling relation. Vertical dotted line in (a) and (b) indicates the time scale of transition from Regime I to Regime II. Dashed line indicates the theoretical fit to empirical data in the range of $\Delta t \leq 1000$ trading days.

Table 1
Parameters from Non-linear fitting of rank mobility data for time scale $\Delta t \leq 1000$ trading days (Regime I in Fig. 2a).

	Estimate	Standard error	*p* value
α	5.9e-03	1.71e-05	<2e-16***
β	4.6e-01	6.03e-04	<2e-16***

Note: *** denotes significance at 0.1% level.

plot in Fig. 2b.

Since the scaling relation of average rank mobility exists, the change of $\overline{RM}$ is very fast at the beginning with the increase of Δt. Then the growth rate gradually decreases, and finally maintains stability. In China's stock market, there is around 250 trading days per years. The finding demonstrates the presence of scale-invariant behavior and long-range temporal organization on rank mobility variations on time scale from one trading day to more than 4 years, and indicates that the memory of all current stock price ranking orders will not fully fade within the next 4 years.

3.3. Robustness of scaling relation on sampling period

To test the robustness of the scaling relation between rank mobility and time scales, the empirical curves in the growth Regime I (Fig. 2a) are presented and analyzed year by year (from 2001 till 2016). We plot all $\overline{RM}$ temporal profiles on a double logarithmic plot. All $\overline{RM}$ profiles exhibit a consistent power-law relation for each consecutive year (straight parallel lines on log-log plot) with power-law exponent $\beta = 0.46 \pm 0.06$ (see Fig. 3). Such persistent power-law behavior indicates a very robust and universal structural organization of the whole market and gives us a reliable way to estimate the future changes of the whole stock price ranking order, which will be helpful for us to evaluate the price volatility of the market and determine the appropriate investment strategy.

4. Conclusion

In this letter, we put forward rank mobility of stock prices and use it to measure the average variation of stock price ranking orders within a certain time interval. Rank mobility reveals the interactions among stocks and the market organization. We study the rank mobility of stock closing prices for a large group of 503 companies registered on the Chinese stock market over a long period of 16

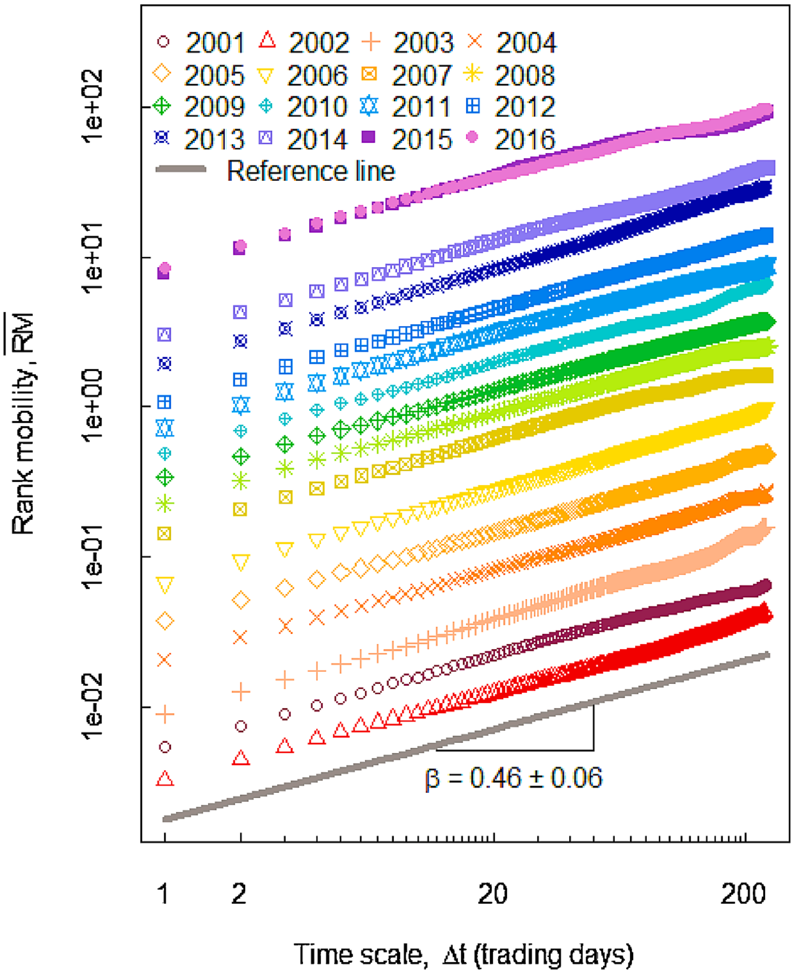

Fig. 3. Power-law relation between rank mobility $\overline{RM}$ and the time scale of observation Δt for 16 different calendar years. Parallel scaling curves, with similar exponent $\beta = 0.46 \pm 0.06$, indicate a robust scaling law. For clarity, a fixed vertical distance of 0.5 is embedded between any two consecutive scaling curves.

years. Specifically, we examine how rank mobility depends on the time scale. We find that the dependence of rank mobility on the time scale is characterized by a power law, indicating a unique scale-invariant organization of stock ranks over a broad range of time scales up to 1000 trading days. Further, this scaling law appears to be robust as it remains stable for consecutive calendar years. Such a robust scaling law in rank mobility provides a new holistic perspective to quantitatively assess and better understand underlying stock market dynamics, and the impact of market rules on rank mobility. Furthermore, our findings may indicate a kind of informational inefficiency of China's A-share market so the long memory in price ranking orders has the potential to be employed in portfolio management and risk management for the investors.

CRediT authorship contribution statement

Yongbin Shi: Methodology, Software, Visualization, Writing - original draft. **Miao Yu:** Visualization, Writing - original draft. **Liujun Chen:** Formal analysis, Project administration. **Plamen Ch. Ivanov:** Supervision, Writing - original draft. **Yougui Wang:** Conceptualization, Writing - review & editing.

Declaration of Competing Interest

No potential conflict of interest was reported by the authors.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.frl.2020.101516.

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